Practice exercises
Given: Friday Oct 19 2001

1. Translate each of the following sentences into the language of propositional calculus using the indicated letters for atomic sentences. For each letter (propositional variable), explicitely define the English statement it represents.

   (1) Deductive logic does not provide a method for establishing the empirical truth of the premises (P).

   (2) Not only is the dolphin air-breathing but it is viviparous as well. (A,V)

   (3) Society must eliminate narrow-minded fundamentalism or accept an ethical absolutism imposed by a small vocal group. (E,A)

   (4) John attends the ballet whenever he has the money for the ticket. (A,H)

   (5) Tim listens to Ricky Martin only if he isn't uninterested in Latin music. (L, I)

   (6) If Stalin was a good Christian, then I'm a monkey's uncle. (S,U)

   (7) A person might become paranoid if not able to function well in long-term situations. (B,F)

   (8) Winning the Nobel Prize in Chemistry is sufficient for you to get a job in the UWO Chemistry Department. (W, J)

   (9) Passing the logic course is a necessary condition for a student to obtain a Computer Science degree. (L,P)

   (10) This element is gold if and only if its atomic number is 79. (G,A)

2. Show by using mathematical induction on $n \in \mathbf{N}$ that the following claim is true:

   If $n \geq 2$ people stand in line and if the first person is a man and the last person is a woman, then somewhere in line there must be a man directly in front of the woman.

Clearly indicate all the steps in the induction proof that is, the induction base, induction hypotesis and induction step, and the conclusion you draw from there.

3. Far away in the warm waters of the Pacific Ocean there is a small forgotten island called TUFA Island. There are two races of people on TUFA island: the Truth Tellers (called TUs) whose statements are always true, and the Falsehood Tellers (called FAs) whose statements are always false.

   This year, alas, the TUFA island has been discovered. On a visit to the island, a CNN reporter spoke to three natives whom he called Blacky, Greeny and Pinky because of their hair colour (and because he could not pronounce their real names). The natives each made the following statements:

   Blacky: *Greeny and Pinky do not belong to the same race.*

   Greeny: *Blacky is a TU.*

   Pinky: *Blacky is a FA.*

   The CNN reporter could not determine from this interview to which race each native belongs. Can you? Justify your answer.

4. Give a truth table for each of the following formulas:

   (1) $A \to (B \to \neg A)$

   (2) $(A \wedge B \to C) \leftrightarrow (A \to C) \vee (B \to C)$

   (3) $A \vee \neg B \to C \wedge \neg A$

   (4) $A \wedge \neg A \to B \vee \neg C$

   (5) $(A \wedge \neg B \wedge C \wedge D) \vee (A \wedge B \wedge \neg C \wedge \neg D)$

5. For the following three arguments, each consisting of some premises and one conclusion

a) Translate the arguments in the language of propositional calculus by using for the propositional variables the letters given in parantheses. Clearly indicate what statement each propositional variable denotes.

b) Determine whether or not the argument is valid (sound) by using:

   For (1), the method that uses tautologies to verify sound reasoning.
   For (2), the method that uses contradictions to verify sound reasoning.

For (3), by showing that the conclusion is a tautological consequence of the premises.

c) Clearly justify any conclusion you draw from a truth table. In general, clearly justify your answers.

1) – If knowing is a state of mind (like feeling a pain), then I could always tell by introspection whether I know.

– If I could always tell by introspection whether I know, then I'd never mistakenly think that I know.

– I sometimes mistakenly think that I know.

Therefore, knowing isn't a state of mind.

[Use S, I and M]

2) – If we had a clear knowledge of God, then our will would be irresistibly attracted to do right.

– If our will were irresistibly attracted to do right, then we'd have no free will.

Therefore, if we had a clear knowledge of God, then we'd have no free will.

[Use G, I and F. From Immanuel Kant)

3) – If you hold a moral belief and don't act on it, then you're inconsistent.

– If you're inconsistent, then you're doing wrong.

Therefore, if you hold a moral belief and act on it, then you aren't doing wrong.

[Use M, A, I and W]

6. At a famous murder trial, four witnesses, Arthur, Betty, Charles and Dorothy, testify under oath.

Determine whether the testimonies are consistent, i.e., whether the set of obtained formulae is satisfiable.

Arthur: "Either Dorothy is innocent or Betty is guilty"

Charles: "Arthur is innocent or Dorothy is guilty, and Betty is innocent if

and only if Arthur is guilty".

Betty: "Either Charles is innocent or Dorothy is guilty"

Dorothy: "Arthur is innocent if and only if either Betty or Charles is innocent"

Note: Whenever translating the English word "or" use "$\vee$" that is, inclusive or.

Note: In your translation, use the letter A to denote the sentence "Arthur is innocent", and use the letters B, C and D similarly for Betty, Charles and Dorothy.

For the following **two** arguments,

a) Translate the argument into the language of propositional calculus.

b) Decide whether or not the argument is valid (sound). If valid, give a proof (by contradiction). If invalid, give a justification.

c) Do not use truth tables for b).

7. *Premise 1.* If a belief is proved, then it's worthy of acceptance.

   Premise 2. If a belief isn't disproved but is of practical value to our lives, then it's worthy of acceptance.

   Premise 3. If a belief is proved then it isn't disproved.

   Conclusion. If a belief is proved or is of practical value to our lives, then it's worthy of acceptance.

   [Use P, W, D and V. The second premise is from the pragmatist philosophy of William James.]

8. *Premise 1.* If Socrates escapes from jail, then he's willing to obey the state only when it pleases him.

   Premise 2. If he's willing to obey the state only when it pleases him, then he doesn't really believe what he says and he's inconsistent.

Conclusion. If Socrates really believes what he says, then he won't escape from jail.

[Use E, W, R, and I. From Plato's "Crito". Socrates had been jailed and sentences to death for teaching philosophy. He discussed with his friend whether he ought to escape from jail instead of suffering the death penalty.]

9. a) Use the truth table method to find the disjunctive normal form of the following formula

$$\neg(A \vee B) \leftrightarrow \neg A \wedge C.$$

b) Use the truth table method to find the conjunctive normal form of the formula

$$((\neg A \leftrightarrow B) \vee C) \wedge (\neg B \wedge C).$$

10. (a) Use the laws of propositional calculus to find a formula in disjunctive normal form tautologically equivalent to the formula in exercise (9a). State all the laws of the propositional calculus that you use.

Is your result tautologically equivalent to the formula you obtained by the truth table method? Justify your answer.

b) Use the laws of propositional calculus to find a formula in conjunctive normal form tautologically equivalent to the formula in (9b). State all the laws of the propositional calculus that you use.

Is your result tautologically equivalent to the formula you obtained by the truth table method? Justify your answer.

11. Translate the following argument into the language of propositional logic. Give a formal proof for the validity of the argument, i.e., prove that the conclusion is formally deducible from the premises. Use the propositional variables C, E, W, A and D for your translation. Clearly state what is the English sentence corresponding to each variable. State each rule you use.

1. If I'm coming down with a cold and I exercise, then I'll get worse and feel

awful.

2. If I don't exercise, then my body will suffer from exercise deprivation and

I'll feel awful.

$\models$ If I'm coming down with a cold, then I'll feel awful.

12. Translate the following argument into the language of propositional logic. Give a formal proof for the validity of the argument, i.e., prove that the conclusion is formally deducible from the premises. Use the propositional variables T, F, S and O for your translation. Clearly state what is the English sentence corresponding to each variable. State each rule you use.

    1. Moral judgments express either truth claims or feelings.

    2. If moral judgments express truth claims, then "ought" expresses either a concept from sense experience or else an objective concept that isn't from sense experience.

    3. "Ought" doesn't express a concept from sense experience.

    4. "Ought" doesn't express an objective concept that isn't from sense experience.

    $\models$ Moral judgments don't espress truth claims but do express feelings.

13. Give a formal proof for the following valid argument. State each rule you use:

    Premises: $(O \wedge G) \rightarrow V, V \rightarrow \neg M, \neg J \rightarrow M, M \rightarrow \neg J$

    Conclusion: $G \rightarrow (O \rightarrow J)$

14. Give a formal proof for the following valid argument. State each rule you use:

    Premises: $A \vee B, C \rightarrow \neg A, B \rightarrow D, C \rightarrow \neg D$

    Conclusion:$\neg C$

15. Give a formal proof for the following argument. Use only the 11 rules of formal deduction and the theorems proved in class by using formal deduction. If you want to use any other theorem, attach a formal proof for it. State each rule you use.

    Premises: $A \wedge (B \vee C)$

    $A \wedge B \rightarrow D \wedge \neg F$

    $A \rightarrow (C \rightarrow \neg(D \vee \neg F))$

    Conclusion: $D \leftrightarrow \neg F$

16. Give a formal proof for the following argument. Use only the 11 rules of formal deduction and the theorems proved in class by using formal deduction. If you want to use any other theorem, attach a formal proof for it. State each rule you use.

Premises: $A \to (B \to C)$

$\neg A \to D \wedge \neg E$

$A \wedge B \to \neg C$

$D \to F \vee G$

$\neg B \to (G \to H)$

$E \vee (H \vee \neg G)$

Conclusion: $G \to H$

17. Consider the connective XOR (exclusive OR) given by:

p	q	p XOR q
0	0	0
0	1	1
1	0	1
1	1	0

Determine whether the set $S = \{\wedge, XOR\}$ is an adequate set of connectives.

18. a) Consider the following system of formal deduction *FDa* that has only two rules of formal deduction:

(i) $A \vdash A$ (Reflexivity)

(ii) If $\Sigma \vdash A$ and $\Sigma \vdash \neg A \vee B$ then $\Sigma \vdash B$ (Disjunctive syllogism)

for all well-formed formulas A, B and all nonempty finite sets Σ of well-formed formulas.

Show that the system *FDa* of formal deduction is sound but not complete.

b) Consider the following system of formal deduction *FDb* that has the rules of formal deduction:

(i) $A \vdash B$

(+) If $\Sigma \vdash A$, then $\Sigma, \Sigma' \vdash A$ (Addition of premises)

($\rightarrow$ $-$) If $\Sigma \vdash A \rightarrow B$ and $\Sigma \vdash A$, then $\Sigma \vdash B$ ($\rightarrow$ elimination)

($\rightarrow$ $+$) If $\Sigma, A \vdash B$, then $\Sigma \vdash A \rightarrow B$ ($\rightarrow$ introduction)

for all well-formed formulas A, B and all finite sets Σ, Σ' of well-formed formulas.

Show that the system of formal deduction *FDb* is not sound but it is complete.

19. Translate the following in the language of Predicate Calculus using:

$C(x, y)$="x caused y"

E(x)="x is evil"

a="Aristotle"

g ="God"

a) Aristotle caused nothing evil.

b) Everything is caused by something or the other.

c) Something evil caused all evil things.

d) God caused everything besides herself.

e) Either someone besides Aristotle is evil, or Aristotle alone is evil.

20. Translate each of the following formulas, of the Predicate Calculus into idiomatic English, using the given interpretation for the constants, function symbols and predicate symbols:

a=Alex

n=Nancy

P(x,y): "x is a problem on exam y"

E(x) : "x is an exam"

M(x): "x is a man"

W(x): "x is a woman"

S(x,y): "x solves y"

a) $\exists x(E(x) \wedge \forall y(P(y, x) \rightarrow S(a, y))$

b) $\forall x(E(x) \wedge \exists y(P(y, x) \wedge S(n, y))$

c) $\exists x \exists y(E(x) \wedge P(y, x) \wedge S(n, y)) \rightarrow \exists u \exists v(E(u) \wedge P(v, u) \wedge S(a, v))$

d) $\exists x(W(x) \wedge \forall y \forall z(E(y) \wedge P(z, y) \rightarrow S(x, z)))$

e) $\forall x(M(x) \rightarrow \exists y \exists x(E(y) \wedge P(z, y) \wedge \neg S(x, z)))$

21. Translate each of the following sentences into the language of Predicate Calculus using only predicate symbols, constants, and function symbols from the list given below. The universe of discourse is the set of integers.

$x > y$: "x is greater than y"

$x \geq y$: "x is greater than or equal to y"

$x = y$: "x equals y"

$x + y$: "the sum of x and y"

$x * y$: "the product of x and y"

Also use the standard symbols for integers:

"- 1" for "negative one", "0" for "zero", etc. You may also use "$x \neq y$" for "x is not equal to y".

a) Every integer has some additive inverse.

b) Not all integers different from zero have a multiplicative inverse.

c)The sum of two odd integers is an even integer.

d) The product of any integer with an even integer is an even integer.

e) Some odd integer is a factor of every even integer.

22. Same for:

a) Every even integer is a multiple of some odd integer.

b) There are two integers that have the same parity, and each is a multiple of seven.

c) Not every pair of integers that has the same parity is a multiple of three.

d) Every pair of distinct integers that has the same parity has the property that their squares also have the same parity. (Two quantities are *distinct* if and only if they are not equal).

e) Each pair of distinct integers that has the same parity has an integer between them.

23. Translate into the language of Propositional Calculus the following statements using:

R(x) : "x is rich"

$x = y$: "x equals y"

b : "Bill Gates"

a) Someone besides Bill Gates is rich.

b) Bill Gates alone is rich.

c) At least two people are rich.

d) Exactly one person is rich.

e) Exactly two persons are rich.

24. For each of the following formulas, use your imagination to find an interpretation and a domain in which the formula is true, and ones in which it is false. Justify your answers.

a) $\forall x(P(x) \rightarrow Q(x)) \vee \exists x(P(x) \wedge \neg R(x))$

b) $\exists x(P(x) \wedge Q(x,a)) \rightarrow \forall y(P(y) \wedge R(y) \rightarrow Q(b,y))$

c) $\forall x R(x, f(x)) \wedge \forall x \forall y(R(x,y) \rightarrow R(y,x)) \wedge \forall x \forall y \forall z(R(x,y) \wedge R(y,z) \rightarrow R(x,z))$

d) $\forall x R(x,x) \vee \forall x \forall y(R(x,y) \rightarrow R(y,x)) \vee \forall x \forall y \forall z(R(x,y) \wedge R(y,z) \rightarrow R(x,z))$

e) $\forall x \forall y(R(x,y) \vee R(y,x)) \wedge \forall x \exists y(R(x,y)) \wedge \forall y \exists x \neg R(x,y)$

25. Prove that:

a) $\forall x(A(x) \wedge B(x)) \models\!\!\!\not\models \forall x A(x) \wedge \forall x B(x)$

b) $\exists(A(x) \vee B(x)) \models\!\!\!\not\models \exists x A(x) \vee \exists x B(x)$

Justify your answers.

26. Prove that it is not the case that:

a) $\exists x A(x) \wedge \exists x B(x) \models\!\!\!\not\models \exists x(A(x) \wedge B(x))$

b) $\forall x A(x) \vee \forall x B(x) \models\!\!\!\not\models \forall x(A(x) \vee B(x))$

Justify your answers. Compare these answers with the answers in Question 2. Give an (informal and intuitive) explanation of the difference.

27. Translate the following argument in the language of predicate calculus and then verify whether or not is correct. Justify your answers. Recall that an argument is correct (valid, sound) if the conclusion is a logical consequence ($\models$) of the premises.

Premise: Everything depends on something or other.

Conclusion: There is something that everything depends on.

Use D(x,y) for "x depends on y".

Some great minds used this type of argument. Aristotle argued "Every agent acts for an end, so there must be some (one) end for which every agent acts." St. Thomas Aquinas argued "If everything at some time fails to exists, then it must be that at some (one) time everything fails to exists." And John Locke argued: "Everything is caused by something, so there must be some (one) thing that caused everything".

28. Translate the following premises (from Lewis Carroll, author of "Alice in Wonderland") in the language of predicate calculus choosing an universe of discourse and appropriate symbols for constants, variables, predicates, or functions. Find out by reasoning intuitively what conclusion you can draw. Prove then that your conclusion really is a logical consequence ($\models$) of the premises. Justify your answers.

Premise 1. Nobody, who really appreciates Beethoven, fails to keep silence while the Moonlight-Sonata is being played.

Premise 2. Guinea-pigs are hopelessly ignorant of music.

Premise 3. No one, who is hopelessly ignorant of music, ever keeps silence while the Moonlight-Sonata is being played.

29. Translate the following valid argument into the language of Predicate Calculus, using the indicated constants and predicate symbols, and give a formal proof.($\vdash$)

"All cats have whiskers. Puffy does not have whiskers. Therefore, Puffy is not a cat." (C(x), W(x), P)

The universe of discourse is the set of all creatures.

30. Translate the following valid argument into the language of Predicate Calculus, using the indicated constants and predicate symbols, and give a formal proof. ($\vdash$)

"All ghosts who live in the old mansion like Oscar. Oscar does not like anyone who likes him. Therefore, any ghost whom Oscar likes does not live in the old mansion." (G(x), M(x), L(x,y), O)

The universe of discourse is the set of all creatures and ghosts.

31. Give a formal proof ($\vdash$) of the following argument:

    Premises: 1. $\forall x(G(x) \rightarrow S(x))$
    2. $G(a)$

    Conclusion: $\exists x S(x)$

32. Give a formal proof ($\vdash$) of the following argument:

    Premises: 1. $\forall(A(x) \wedge B(x) \rightarrow D(x))$
    2. $\exists x(A(x) \wedge B(x) \wedge C(x))$

    Conclusion: $\exists x(A(x) \wedge C(x) \wedge D(x))$

33. Give a formal proof ($\vdash$) of the following argument:

    Premises: 1. $\forall x[P(x) \rightarrow \forall y(Q(y) \rightarrow \neg R(x,y))]$
    2. $\forall x[P(x) \rightarrow \forall y(\neg S(x) \rightarrow Q(y))]$
    3. $P(b)$

    Conclusion: $\exists x[P(x) \wedge \forall y(R(x,y) \rightarrow S(x))]$

34. Use resolution with the set of support strategy to show that the following argument is valid:

    Premise 1: $P \rightarrow Q$

    Premise 2: $\neg P \rightarrow R$

    Premise 3: $Q \vee R \rightarrow S$

    Conclusion: S

    Describe in details all the steps involved. Explain why the result of the resolution alghorithm imply the validity of the argument.

35. Consider that by adding the negation of the conclusion to the set of premises we obtained the set S of clauses:

$\{P, \neg R\}, \{Q, \neg R\}, \{Q, \neg S\}, \{\neg P, T\}$
$\{\neg Q, \neg T\}, \{\neg Q, R, T\}, \{P, S, \neg T\}, \{\neg P, Q, R\}$
$\{Q, R, S, T\}$

Apply the Davis-Putnam procedure to show that the set of clauses is not satisfiable. Explain how the outcome of the Davis-Putnam procedure implies unsatisfiability.

Show in detail all the intermediary steps. In particular, for each elimination of a variable, show which are the sets S_i, S_i', T_i and U_i. For each resolvent indicate which are the parent clauses. Eliminate the variables in the order P,Q,R,S.

36. Let $D(x), P(x), O(x)$ and $W(x)$ be the statements "x is a duck", "x is one of my poultry", "x is an officer" and "x is willing to waltz". The universe of discourse is the set of all creatures.

Translate the following argument in the language of predicate calculus. Use resolution with the set of support strategy to find out whether or not the argument is correct.

Premise 1: No duckes are willing to waltz.

Premise 2: "No officers ever decline to waltz"

Premise 3: "My poultry are ducks"

Conclusion: "My poultry are not officers"

37. Use resolutuon for predicate calculus to show that the following argument is valid:

Premise 1: "$\exists x[P(x) \wedge \forall y(Q(y) \rightarrow \neg R(x, y))]$"

Premise 2: "$\forall[P(x) \rightarrow \forall y(S(y) \rightarrow R(x, y))]$"

Conclusion: $\forall x(Q(x) \rightarrow \neg S(x))$

Show in details all the steps involved (obtaining of —esists-free prenex normal form, transformation into clauses, resolution procedure).