Example

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The results of a hypothesis testing using an F statistic should (obviously) be independent on the choice of the samples (which one is the numerator and which one is the denominator). This is an example showing this.

The expression level of a gene is measured in a number of control subjects and patients. The values measured in controls are: 10, 12, 11, 15, 13, 11, 12 and the values measured in patients are: 12, 13, 13, 15, 12, 18, 17, 16, 16, 12, 15, 10, 12. Is the variance different between controls and patients?

The null hypothesis is: $H_0 : \sigma_A^2 = \sigma_B^2$ and the research hypothesis is: $H_a : \sigma_A^2 \neq \sigma_B^2$. We know that the ratio of the sample variances of two samples drawn from two normal populations with the same population variance will follow an F distribution.

We consider sample 1 coming from population A and sample 2 coming from population B: $\nu_A = 6$, $s_A^2 = 2.66$, $\nu_B = 12$, $s_B^2 = 5.74$.

We choose to consider the ratio s_A^2/s_B^2 first. Therefore, we use:

$$F_{0.975(6,12)} = 3.73 \tag{1}$$

$$F_{0.025(6,12)} = \frac{1}{F_{0.975(12,6)}} = \frac{1}{5.37} = 0.1862 \tag{2}$$

If the statistic s_A^2/s_B^2 is lower than 0.186 or higher than 3.73 we will reject the null hypothesis. If the statistic s_A^2/s_B^2 is in between the two values, we cannot reject the null hypothesis:

$$\frac{1}{5.37} \leq \frac{s_A^2}{s_B^2} \leq 3.73 \tag{3}$$

Now, let us consider the ratio s_B^2/s_A^2.

$$F_{0.975(12,6)} = 5.37 \tag{4}$$

$$F_{0.025(12,6)} = \frac{1}{F_{0.975(6,12)}} = \frac{1}{3.73} = 0.268 \tag{5}$$

If the statistic s_B^2/s_A^2 is lower than 0.268 or higher than 5.37 we will reject the null hypothesis. If the statistic s_B^2/s_A^2 is in between the two values, we cannot reject the null hypothesis:

$$\frac{1}{3.73} \leq \frac{s_B^2}{s_A^2} \leq 5.37 \tag{6}$$

An examination of equations 3 and 6 reveals that the same conditions are used in both cases, independently of the choice of the populations. If s_A^2 is to be significantly higher than s_B^2, the statistic s_A^2/s_B^2 has to be higher than $F_{0.975(6,12)} = 3.73$ or, alternatively the statistic s_B^2/s_A^2 has to be lower than $F_{0.025(12,6)} = \frac{1}{F_{0.975(6,12)}} = \frac{1}{3.73}$.