Computer Communications 24 (2001) 1329–1340
computer communications
www.elsevier.com/locate/comcom

Robust control and rate coordination for efficiency and fairness in ABR traffic with explicit rate marking ☆

L. Schwiebert*, L.Y. Wang

Department of Electrical and Computer Engineering, Wayne State University, Detroit, MI 48202-3902, USA

Received 14 March 2000; accepted 28 November 2000

Abstract

The problem of efficiency and fairness of Available Bit Rate (ABR) traffic with Explicit Rate (ER) marking is investigated from a control point of view. It is revealed that due to significant uncertainty on time delays in ABR traffic and strong interaction among switches, ABR traffic imposes a great challenge to control strategy development. Focusing on robustness of congestion control and asymptotic fairness, we propose a new control strategy that employs robust control to accommodate delay uncertainty and rate coordination for fairness. Interaction between these two control actions leads to a non-standard multi-objective control problem. Overall stability of the system and convergence to fairness are rigorously analyzed. Conditions are presented under which robust stability and convergence are guaranteed. Simulation tests are then performed to demonstrate that a redesign of feedback controllers may become necessary to satisfy these conditions. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: ATM networks; Available bit rate; Congestion control; Fairness coordination

1. Introduction

In this paper, the problem of efficiency and fairness of Available Bit Rate (ABR) traffic with ER marking is investigated from a control point of view. Taking typical network environments into consideration, we focus on the generic features, essential issues, and potential control strategies arising in such problems. It is revealed that due to significant uncertainty on time delays in ABR traffic and strong interaction among switches, ABR traffic imposes a great challenge to control strategy development. Focusing on robustness of congestion control and asymptotic fairness, we propose a new control strategy that employs robust control methodologies to deal with uncertain time delays on connections and switches, and performs rate coordination to provide fairness among connections while achieving high efficiency.

1.1. ABR traffic

The ATM Forum Traffic Management Specification Version 4.1 [2] defines various ATM service categories. Among these, ABR service was created for data communications that are relatively delay-insensitive but are sensitive to data loss.

As the name implies, ABR connections receive the bandwidth that has not been assigned to constant bit rate (CBR) connections and is not currently used by variable bit rate (VBR) connections. The Quality of Service (QoS) guarantees for ABR connections provide that data can always be sent at some minimum rate (perhaps zero) known as the minimum cell rate (MCR). In addition, the connection also has a maximum transmission rate, known as the peak cell rate (PCR), that cannot be exceeded. The available bandwidth must be allocated “fairly” among the ABR connections. The actual amount of bandwidth an ABR connection receives depends on information exchanged among the source, destination, and switches along the path from the source to the destination. This information is provided to the source using rate-based closed-loop feedback control. Typically, feedback is provided in specific fields of the Resource Management (RM) cells that are periodically transmitted by the source for the express purpose of obtaining feedback. The algorithm employed by the switch to provide this feedback is implementation-dependent, but uses one of three techniques: Explicit Forward Congestion Indication (EFCI) marking, relative rate marking, or ER marking.

☆ This research is supported in part by NSF Grant ECS-9634375.

* Tel.: +1-313-577-5474; fax: +1-313-577-1101.
E-mail addresses: loren@ece.eng.wayne.edu (L. Schwiebert), lywang@ece.eng.wayne.edu (L.Y. Wang).

PII: S0140-3664(01)00287-0

Switches that use EFCI marking set a bit in the header of one or more *data* cells to indicate that the switch is congested. This technique was used in first generation switches before the RM cell specifications were finalized. There are severe limitations with this approach, among which is the "beat-down" effect, where connections that traverse many switches are less likely to receive their fair share of the bandwidth than connections that traverse few switches. The second technique, relative rate marking, uses two fields in the RM cell — the Congestion Indication (CI) bit and the No Increase (NI) bit. The CI bit is set by the switch or destination to indicate that the source should reduce its transmission rate and the NI bit is set to indicate that the source should maintain its current transmission rate. If neither bit is set, the source is permitted to increase its transmission rate. The rate adjustments are made using: (a) additive increase, based on a value less than one, called the Rate Increase Factor (RIF); and (b) multiplicative decrease, based on a value less than one, called the Rate Decrease Factor (RDF). This coarse-grain control constrains the responsiveness of the system. Consequently, relative rate marking can avoid the beat-down problem inherent in EFCI marking, but can suffer from slow response to congestion as well as oscillation even at steady state. This oscillation can reduce network utilization and fairness.

ER marking, which is the scheme of choice in most current ATM switches, uses the ER field of the RM cell to specify an exact rate at which the source should send data. This scheme, although more complex in implementation than the other two schemes, provides more precise information to the source and enhances flexibility in designing control algorithms. As a result, it has gained tremendous popularity in ABR rate control. The ATM Traffic Management Specification [2] has provided a mechanism for circumventing complications arising from inter-operating relative rate marking and ER marking by default initialization of RIF and RDF to suitable values, paving the path to more practical implementations of ER marking strategies.

1.2. Main issues and basic approaches

ABR traffic imposes a great challenge to control strategy development, partially due to the following factors:

1. Delay times of network channels differ greatly among different connections, possibly by orders of magnitude. Delays also vary with time, due to changes in network traffic conditions.
2. Fairness is a global property of ABR networks and requires coordination among switches and sources. On the other hand, global information on fairness is not available to individual switches since this information is maintained in a distributed manner.
3. The goal of congestion control at switches for maximizing network utility and minimizing cell loss, and the goal of fairness often conflict when network conditions are changing. The assumption that one can wait until network traffic settles to steady state before implementing fairness algorithms is not realistic, because steady state may not be reached frequently due to dynamic demands and bandwidth allocations. Hence, a satisfactory tradeoff between congestion control and controlling for fairness must be sustained dynamically.
4. ABR traffic relies on available bandwidth. Variations in available bandwidth, which represent disturbances to the system, can be large and unpredictable.

Our approach to dealing with these issues can be summarized as follows: robust controllers are first designed for local queue control.[1] The design guarantees local robust stability, and fast response of sending rates to variations in available bandwidth in the presence of unknown channel time delays. Second, a fairness algorithm is devised which calculates desired bandwidth distribution for each connection with fairness as its objective function. During transient network conditions, the actual control action is a weighted combination of queue control and fairness control. The weighting function is then adapted based on an indicator of network traffic conditions. The indicator reflects essentially the speed of traffic pattern changes. When the traffic pattern is changing dramatically, the control action is more inclined to queue control to maintain system stability and tracking performance. When the traffic pattern is relatively smooth, fairness coordination becomes more dominant. If certain conditions on traffic patterns, and the appropriate selection of coordination mechanisms are met; we show that this coordination leads to a stable system with asymptotic fairness, fast response, and robustness.

1.3. Related work

The feedback structure of ABR network systems strongly suggests a potential utility of feedback control theory in analyzing system behavior and designing efficient and robust network control algorithms. Substantial efforts have been devoted to this pursuit by researchers from the communications, control, and computer communities. For instance, linear control theory has been applied to the analysis of ATM network traffic management problems [11,15,20].

Although the standard queue control problems conform to typical feedback control structures, fairness requirements mandate system coordination and profoundly complicate the control problem. A number of switch algorithms have been developed to provide a fair share of the available bandwidth to each ABR connection. A non-exhaustive list includes [3–8,12,13,16]. The primary objectives of these algorithms are high utilization and ensuring that each ABR connection receives a fair share of the bandwidth currently available at the switch. Max–min fair allocation

[1] We use the term queue control instead of congestion control in the rest of the paper, since from a control perspective, we are achieving congestion control by controlling the queue.

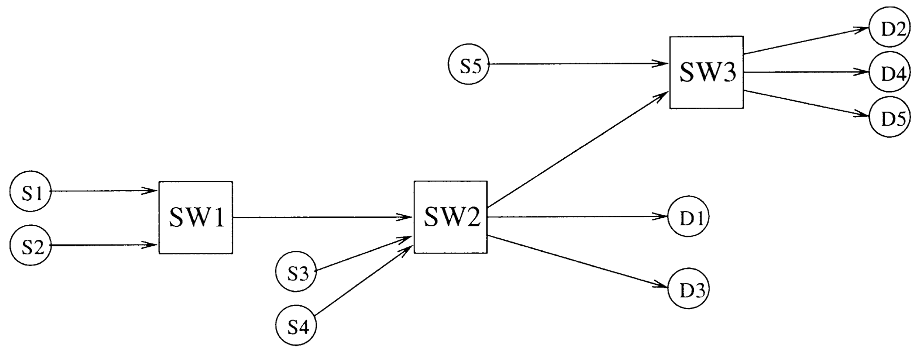

Fig. 1. A typical ABR network.

[9] is the measure of fairness most often used by the algorithms. Using this metric, each connection should receive as much bandwidth as possible, with the constraint that a connection should not be allocated bandwidth that could be used by another connection with less bandwidth. Since the switches do not have global knowledge of the network traffic and the available bandwidth on the links, one must resort to local information to devise fairness tuning algorithms, leading to a difficult control task for achieving a fair partition of available bandwidth among competing connections.

Many studies of various rate marking algorithms and bandwidth allocation schemes have relied primarily or exclusively on simulation results to justify their performance. Although such approaches are of value in evaluating complicated network systems, to gain insights on fundamental issues involved in ATM rate control problems it is desirable and essential to perform rigorous analysis, at least on simplified systems. Recent work on ATM rate control using game-theoretic approaches [1] and on network resource allocation using optimization methodologies [17] are representative of this direction.

Most recent papers on ABR rate control employ more or less standard results from basic control tools, such as linear feedback [15], pole placement [11], etc. By using discrete-time models, the fundamental problem of stability under varying delays (infinite dimensional systems) is avoided. Furthermore, other than simulation demonstrations, evaluation of the interaction between queue control and fairness is largely unexplored.

In this paper, it is demonstrated that the control issues inherent in ABR traffic are both non-standard and non-trivial. Rigorous analysis of stability and convergence to an efficient and fair operating point is then carried out for ABR rate control systems in the presence of variable time delays and including the interaction between queue control and fairness.

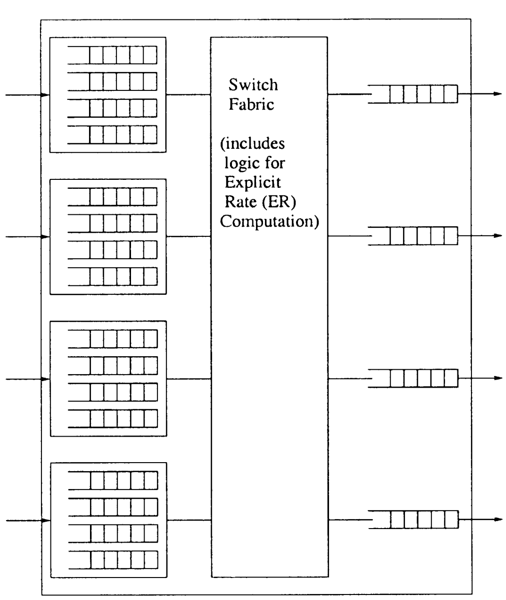

Fig. 2. A generic 4×4 switch.

2. Problem formulation

An ATM network consists of sources, switches, and destinations, such as the one in Fig. 1. Due to variations in other higher priority network traffic, such as CBR and VBR, an ABR connection experiences significant uncertainty on the available bandwidth during its operation. In principle, it is highly desirable that sources can adjust their sending rates promptly to accurately track their assigned bandwidth so that the network utility can be maximized without cell loss.

Here, sources $S1$ through $S5$ transmit data via different routes which consist of various numbers of switches. A typical switch usually contains input buffers, a switch fabric or internal bus, and output buffers. Depending on system configurations, the switch may maintain multiple input and output queues. Focusing on a detailed model of switch hardware designs will obscure the fundamental issues involved in traffic problems, so a generic switch hardware architecture [10] is assumed. An example of a 4×4 switch is shown in Fig. 2. This switch uses four virtual (logical) queues for each input port, one of which maps to each of the four output ports. The switch fabric controls the transfer of cells from the input ports to the output ports based on the queuing discipline. The switch fabric also contains the logic for computing the rate locally available to each connection and fed back in the RM cells.

To capture the generic features of ABR traffic problems, we assume in this paper that at the time of connection establishment the switch sets up a virtual buffer for internal routing of each connection's ATM cells. The data stored in the buffer forms a queue which smoothes out differences between the input and output rates of data transmission. A queue with a large number of cells consumes extra memory and creates time delays in data communication. In the worst case of the queue size exceeding the available buffer space, cell loss will occur. On the other hand, a small queue will drain rapidly when the output bandwidth exceeds the input rates, resulting in under-utilization of the available bandwidth and hence reduced efficiency. This phenomenon becomes more pronounced when connections are subjected to large time delays and available bandwidth varies dramatically. Thus, one goal is to maintain a queue size that is modest, but not zero.

2.1. Models

An ATM network can be modeled in either continuous-time or discrete-time. Although data transmission is physically executed in discrete-time, the nature of high-speed data transmission renders continuous-time models a viable representation. In fact, it offers several advantages over discrete-time models. First, switches and host computers may have different clocks. Mandating a universal sampling interval for the whole network, as assumed quite commonly in the literature, is unrealistic. Second, time delays on connections are not a multiple of sampling intervals. Using continuous-time models avoids approximating time delays. Third, it is well known that stability and performance of the discrete-time system does not automatically imply these properties hold for the modeled system. For these reasons, this paper will employ continuous-time models.

To model the ABR rate control problem in a formal way, a number of mathematical expressions with associated notation are introduced. The basic symbols used in this paper are summarized in Appendix A.

2.1.1. Switches and connections

Suppose that at time t, a typical switch, SW, has n input ABR connections and the same number of output connections. Denote these immediate sources, which may be a source or another switch, by $S_1, \ldots, S_n$.

Let $r_i(t)$ be the sending rate of source S_i at time t, for $i = 1, \ldots, n$. Due to network delays, the arrival rate of the ith connection, $x_i(t)$, at switch SW is related to r_i by

$$x_i(t) = r_i(t - \tau_i^{\mathrm{fw}}),$$

where τ_i^{fw} is the time delay for connection i in the forward direction. The output rates at SW will be $d_1, \ldots, d_n$, which may go to a destination or another switch.

Although hardware configurations may differ, for analysis of system behavior one can lump the effects of internal buffers into one virtual buffer for each connection. The queue size of the ith connection at time t is $q_i(t)$. The dynamic equation of the queue for the ith connection at the switch is simply

$$\dot{q}_i(t) = x_i(t) - d_i(t).$$

To maintain high network utility and low cell loss ratios, one designates a desired queue size q_i^0. q_i^0 is determined on the basis of switch structures, buffer sizes, delays on connections, typical traffic patterns, and variations in available bandwidth.

The first goal of ABR rate control is to ensure that $q_i(t)$ is close to q_i^0, namely reducing the error

$$e_i = q_i - q_i^0.$$

The error dynamics are

$$\dot{e}_i(t) = \dot{q}_i(t) = x_i(t) - d_i(t) = r_i(t - \tau_i^{\mathrm{fw}}) - d_i(t).$$

The input–output relationship of the ith connection can also be expressed in the frequency domain as

$$E_i(s) = \frac{1}{s}(\mathrm{e}^{-\tau_i^{\mathrm{fw}} s} R_i(s) - D_i(s)),$$

where $E_i(s)$, $R_i(s)$, and $D_i(s)$ are the Laplace transforms of e_i, r_i, and d_i, respectively.

2.1.2. Control

The control action u_i, the ER for the ith connection sent from the switch, is generated at the switch based on a certain design methodology defined later in the paper. Structurally, u_i may be designed as a function of e_i (feedback), d_i (feedforward), and other traffic information (coordination)

$$u_i = u_i^{\mathrm{T}} + u_i^{\mathrm{c}},$$

where $u_i^{\mathrm{T}} = f_i(e_i, d_i)$ is the component for queue control and u_i^{c} is the component for coordination with other switches for fairness.

If one is to use linear time-invariant controllers for queue control, u_i^{T} will have the structure

$$U_i^{\mathrm{T}}(s) = -F_i(s)E_i(s) + K_i(s)D_i(s),$$

in its transfer function form. It is well known that although the feedback controller F_i is allowed to be unstable, the feedforward control K_i must be stable.

The ER u_i is then sent back to the sending source S_i via the RM cell. The sending rate $r_i(t)$ is adjusted according to u_i after a delay τ_i^{bw} along the backward path. Then, $R_i(s) = \mathrm{e}^{-\tau_i^{\mathrm{bw}} s} U_i(s)$. We will use $\tau_i = \tau_i^{\mathrm{fw}} + \tau_i^{\mathrm{bw}}$ to denote the round-trip delay time. It follows that:

$$E_i(s) = \frac{1}{s}(\mathrm{e}^{-\tau_i^{\mathrm{fw}} s} U_i(s) - D_i(s)) = P_i(s)R_i(s) - \frac{1}{s}D_i(s),$$

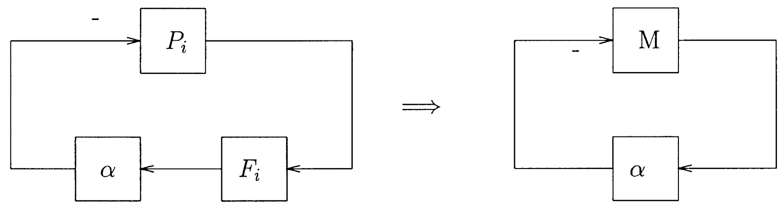

Fig. 3. Closed-loop system for stability analysis.

where $P_i(s) = (1/s)\,\mathrm{e}^{-\tau_i s}$ is the transfer function from u_i to e_i, which is the plant to be controlled.

Without u_i^{c} and other nonlinearities which will be introduced shortly for fairness coordination, upon applying the feedback u_i^{T} the ith connection will have a closed-loop system expressed by

$$E_i(s) = -\frac{1}{s}\,\mathrm{e}^{-\tau_i s}F_i(s)E_i(s) + \frac{1}{s}\,\mathrm{e}^{-\tau_i s}K_i(s)D_i(s) - \frac{1}{s}D(s),$$

or

$$E_i(s) = \frac{\mathrm{e}^{-\tau_i s}K_i(s) - 1}{s + \mathrm{e}^{-\tau_i s}F_i(s)}D_i(s).$$

Also,

$$X_i - D_i = sE_i = \frac{\mathrm{e}^{-\tau_i s}K_i(s) - 1}{1 + \dfrac{1}{s}\,\mathrm{e}^{-\tau_i s}F_i(s)}D_i(s)$$

$$= \frac{\mathrm{e}^{-\tau_i s}K_i(s) - 1}{1 + P_iF_i(s)}D_i(s).$$

2.1.3. Information on unused bandwidth

Although the information on fairness is not available to individual switches, connections that are bottlenecked elsewhere in the network will leave some assigned bandwidth unused at this switch. This information is first sent back to the sources via RM cells and then propagates along the routes to other switches as indicated by lower actual sending rates than the bandwidth assigned at these switches.

Mathematically, if the ith connection is bottlenecked with peak rate c_i, its sending rate can be modeled as a saturation node of value c_i. That is,

$$r_i(t) = \max\{\min\{u_i(t - \tau_i^{\mathrm{b}}), c_i\}, 0\} := S(u_i(t - \tau_i^{\mathrm{b}})),$$

where τ_i^{b} is the time delay from the time u_i is sent from the switch to the time the information of a bottleneck becomes available to the switch. This saturation is the only information available to the switch concerning fair allocation of the available bandwidth. This bottleneck leads to unutilized network resources. As a result, rate coordination will be performed on the basis of this saturation information.

One challenge in employing this information for fairness coordination is that due to significant time delays in data transmission from the original source to the switch, $r_i(t)$ may be temporarily lower than assigned bandwidth even if no bottleneck occurs. This will happen frequently when available bandwidth traffic patterns vary significantly due to external disturbances such as the initiation or termination of higher priority connections. In other words, it is extremely difficult to dynamically identify the causes of lower sending rates. This difficulty will be resolved in the next section by introducing a weighted multi-objective control algorithm.

3. Control structures and strategies

ABR rate control imposes multiple control objectives. First, for each individual switch and route it is desirable that the sending rate matches closely and promptly the assigned bandwidth so that network resources can be optimally utilized. This is essentially a tracking requirement in which sending rates must follow assigned bandwidth. Second, to avoid cell loss and enhance smooth data transmission, certain queue sizes must be maintained. This can be expressed as a regulation and stability problem in which the actual queue size must be controlled within the vicinity of a constant. Finally, to achieve fairness among connections, assigned bandwidths must be adjusted dynamically based on distributions of available bandwidth over the network.

3.1. Stability and tracking in queue control

Queue control at a switch aims at the reduction of e_i (queue stability) and x_i–d_i (error tracking). Since

$$X_i(s) - D_i(s) = sE_i(s),$$

we will first focus on the reduction of x_i–d_i.

Variations in available bandwidth for ABR traffic are confined by physical limitations of channel bandwidth and averaged out by randomness of connections and disconnection. In this paper, we assume that available bandwidth variations are large below a given frequency bound ω_M and small beyond that. Consequently, reduction of x_i–d_i is to be achieved satisfactorily over the frequency band $[0, \omega_M]$. This requirement can be expressed conveniently by defining a stable weighting function, $W(s)$, whose gain spectrum, $|W(\mathrm{j}\omega)|$, is close to one in the band $[0, \omega_M]$ and decreases quickly outside it. As a result, the design problem becomes the reduction of the weighted transfer function

$$|W(\mathrm{j}\omega)(X_i(\mathrm{j}\omega) - D_i(\mathrm{j}\omega))| = \left| W(\mathrm{j}\omega)\frac{\mathrm{e}^{-\mathrm{j}\omega\tau_i}K_i(\mathrm{j}\omega) - 1}{1 + P_i(\mathrm{j}\omega)F_i(\mathrm{j}\omega)} \right|,$$

over all ω. This is equivalent to the reduction of the H^∞ norm of

$$W(s)\frac{\mathrm{e}^{-\tau_i s}K_i(s) - 1}{1 + P_i(s)F_i(s)}.$$

F_i and K_i can be designed by using classical compensation methods such as the classical PID (proportional, integral, and derivative) controller or lead–lag compensation.[2] It is observed that $K_i(s)$ is a feedforward mapping and does not affect stability. It follows that one may select a stabilizing feedback $F_i(s)$ first. For instance, if $F_i(s) = k$, a gain feedback, then the standard Nyquist stability analysis leads to the stability condition

$$k < \frac{\pi}{2\tau_i}.$$

After the selection of $F_i(s)$, $K_i(s)$ can then be designed to reduce or minimize

$$W(s)\frac{\mathrm{e}^{-\tau_i s}K_i(s) - 1}{1 + P_i(s)F_i(s)}.$$

3.2. Fairness

To achieve fairness, one must consider the interactions among the connections. We assume that the information on whether a connection is bottlenecked or not can be obtained after a certain time delay τ_i^{b}. In particular, if the measured rate $x_i(t) = c_i < u_i(t - \tau_i^{\mathrm{b}}) - \delta$ where δ is a threshold value, then we label the ith connection as bottlenecked with saturation value c_i. Due to the time-varying nature of ATM networks, c_i will vary with time.

Assume that the ABR bandwidth of the switch at time t is $B(t)$. At time t, assume that m connections, say, $r_1, \ldots, r_m$, are bottlenecked with saturation values $c_1(t), \ldots, c_m(t)$, respectively. A commonly used algorithm for distributing bandwidth can be described as follows: Let

$$a(t) = \frac{B(t) - (c_1(t) + \cdots + c_m(t))}{n - m}. \quad (1)$$

The fair share functions for the connections are then defined by

$$f_i(t) = c_i(t), \qquad i = 1, \ldots, m; \qquad f_i(t) = a(t), \qquad i = m + 1, \ldots, n. \quad (2)$$

For fairness, it is desirable that $r_i(t)$ remains close to $f_i(t)$ for all t, or at least, asymptotically as $t \rightarrow \infty$. Note that $f_i(t)$ is always bounded by $B(t)$.

It is straightforward to show that at steady state when the available bandwidth remains constant, this locally executed algorithm will lead to a convergence to max–min fairness. The reader is referred to Refs. [8,13] for further details.

However, the main issue here is the interaction between queue control and fairness algorithms. What is the impact of fairness algorithms on the stability of queue control? Will convergence to fairness remain valid when queue control is continuously in operation? How should one decide whether to use queue control or fairness algorithms to generate control actions? These will be answered rigorously in the subsequent sections.

3.3. Coordination of queue control and fairness

The objectives of queue control and fairness coordination often conflict during dynamic transient network conditions, e.g. when the output rates d_i vary significantly with time. Consequently, a tradeoff must be retained in deciding control actions.

Our algorithms for determining the total control actions can be summarized as follows. Let $U_i^{\mathrm{T}}(s) = -F_i(s)E_i(s) + K_i(s)D_i(s)$, $\quad i = 1, \ldots, n$, be the control action based on dynamic queue control of the ith connection. At time t, the total control action, $u_i(t)$, will be determined by

$$u_i(t) = \alpha(t)u_i^{\mathrm{T}}(t) + (1 - \alpha(t))f_i(t), \qquad i = 1, \ldots, n. \quad (3)$$

where $0 \le \alpha(t) \le 1$. The special case $\alpha = 1$ represents control actions for queue control only; the opposite extreme case $\alpha = 0$ indicates pure fairness assignment of available bandwidth.

One of the most important issues here is the selection of the weighting $\alpha(t)$. Intuitively, during dramatic dynamic intervals, the control actions should incline more toward traffic regulation. During relatively smooth operations, the control signals should be tuned toward fair assignments. The OSU Scheme [8] proposes that one specifies a phase in which $\alpha = 1$ and afterwards switches to $\alpha = 0$ for fairness.[3] The main drawback of this approach is that it is very difficult to specify such phases since network traffic seldom settles

---

[2] If one is to pursue optimal solutions, then the optimization problem can be generically expressed as

$$\inf_{F_i, K_i \text{ stabilizing}} \left\| W\frac{\mathrm{e}^{-\tau_i s}K_i - 1}{1 + P_iF_i} \right\|_\infty.$$

After the selection of a stabilizing $F_i(s)$, the design problem is reduced to

$$\inf_{K_i \in H^\infty} \left\| W\frac{\mathrm{e}^{-\tau_i s}K_i - 1}{1 + P_iF_i} \right\|_\infty.$$

By defining $W_0 = (W)/(1 + P_iF_i) \in H^\infty$, we obtain

$$\inf_{K_i \in H^\infty} \|W_0(\mathrm{e}^{-\tau_i s}K_i - 1)\|_\infty = \inf_{K_i \in H^\infty} \|W_0 - W_0\,\mathrm{e}^{-\tau_i s}K_i\|_\infty$$

$$= \inf_{K_i \in H^\infty} \|W_0 - B_iK_i\|_\infty,$$

where $B_i = W_0 e^{-\tau_i s}$. The last expression of the above equation is a standard H^∞ optimal model matching problem for systems with time delay. Since solutions to this problem can be readily obtained by using standard software packages such as Matlab μ-Tools, we will not discuss the details further.

[3] The OSU Scheme is not directly expressed in terms of the framework introduced in this paper. The ideas, however, can be interpreted as such.

down to an equilibrium point [14]. Also, the sudden change from one phase to another may create unnecessary disturbances to the system.

In this paper, we introduce an indicator of traffic pattern variations and use that indicator to tune $\alpha(t)$ smoothly. Generically, the indicator is a function of the output rates $d_i(t)$ and ABR target bandwidth $B(t)$

$$g(t) = G(d_i, B).$$

Typical examples include

$$G(d_i, B) = \frac{b_1 s}{s + b_2} d_i + \frac{c_1 s}{s + c_2} B, \tag{4}$$

where b_1, b_2, c_1, $c_2 > 0$. This will essentially measure the rates of change of d_i and B with high frequency noises filtered. Then

$$\alpha(t) = \frac{|g(t)|}{a + |g(t)|},$$

where $a > 0$ is a scaling factor. Note that in this expression, $0 < \alpha(t) < 1$. Also, $\alpha(t)$ is close to one when $|g(t)|$ is large and close to 0 when $|g(t)|$ is small.

4. Main theoretical results: stability and convergence

In this section, we present the main theoretical results of this paper on stability of the total system and conditions under which convergence to fairness is guaranteed.

The problems addressed here are non-standard and non-trivial. (1) Although one may design F_i and K_i to ensure stability of each individual feedback loop, it is well understood that interaction among subsystems, caused here by fairness coordination, can adversely affect their behavior and lead to instability. This fact seems to be largely overlooked in the literature on ABR rate control. Often, simulation is used to demonstrate stability after interaction takes place. In this paper, rigorous results will be presented. (2) It is clear that one can always use very small gain in the feedback loop to ensure stability. But this reduces efficiency and increases response time. On the other hand, fast tracking favors large gains, which adjusts to changes in available bandwidth more quickly. However, this could lead to instability. Without guidance from theoretical analysis, one is left with parameter tuning and simulation in designing controllers. This paper presents explicit conditions which will guarantee the stability of the total system. These conditions can then be employed to design stable controllers which do not necessarily have very low gains. (3) Establishing convergence properties without assuming steady state operation remains an open and challenging problem. In this paper, we show that by appropriate design of the traffic pattern indicators and coordination of the weighting functions, convergence to fairness can be guaranteed. The conditions are generic and hence can be employed to design different indicators and weight functions.

4.1. Robust stability and redesign of F_i

Although we have established stability when the control action is determined solely on the basis of queue control, the modified strategy (3) potentially leads to instability. In Theorem 1, we provide a condition for redesigning controllers under which stability is guaranteed even when coordination for fairness is used.

The modified control action results in the following feedback system:

$$\begin{aligned} u_i(t) &= \alpha(t) u_i^0(t) + (1 - \alpha(t)) f_i(t) \\ &= \alpha(t)(-F_i e_i + K_i d_i) + (1 - \alpha(t)) f_i(t). \end{aligned} \tag{5}$$

Since α is time varying, we shall view the network and controllers as time-varying systems. They will be still denoted by the plant (the system from u_i to e_i) $P_i = (1/s)\,e^{-\tau_i s}$, feedback F_i, and feedforward K_i, respectively. Also, s, $1/s$, $e^{-\tau_i s}$ will be used to indicate differentiation, integration, and delay systems, respectively.

$$\begin{aligned} E_i &= \frac{1}{s}[e^{-\tau_i s}(\alpha(-F_i E_i + K_i D_i) + (1 - \alpha) f_i) - D_i] \\ &= -P_i(\alpha F_i E_i) + P_i(\alpha K_i D_i) + \frac{1}{s}\, e^{-\tau_i s}(1 - \alpha) f_i - \frac{1}{s} D_i, \end{aligned}$$

and

$$\begin{aligned} E_i &= (1 + P_i \alpha F_i)^{-1}\left(P_i \alpha K_i - \frac{1}{s}\right) D_i \\ &\quad + (1 + P_i F_i)^{-1} \frac{1}{s}\, e^{-\tau_i s}(1 - \alpha) f_i = M_1 D_i + M_2 f_i. \end{aligned}$$

It follows that

$$X_i - D_i = s E_i = s(M_1 D_i + M_2 f_i).$$

Since $0 < \alpha(t) < 1$, for constant values of α stability is maintained. However, when α varies with time, stability analysis becomes more involved.

For stability analysis, define the open-loop system

$$M(s) = \frac{1}{s}\, e^{-\tau_i s} F_i(s) = P_i(s) F_i(s).$$

The closed-loop system under study can then be transformed into the feedback system in Fig. 3.

Theorem 1. *Suppose that the time delay $\tau_i \le \bar{\tau}_i$ and $f_i(t)$ is bounded. If F_i satisfies*

1. *F_i robustly stabilizes P_i for all $\tau_i \le \bar{\tau}_i$,*
2. $$\sup_{0 < \tau_i \le \bar{\tau}_i} \|M(1 + M)^{-1}\|_\infty \le 1,$$

then, the closed-loop system in Fig. 3 *is robustly stable for all time varying $\alpha(t)$ with $0 < \alpha(t) < 1$ and $0 \le \tau_i \le \bar{\tau}_i$.*

Proof. Since F_i stabilizes P_i, the closed-loop systems with $\alpha \equiv 1$

$$(1 + M)^{-1} \text{ and } M(1 + M)^{-1},$$

are stable with

$$\mu = \|M(1 + M)^{-1}\|_\infty < \infty.$$

Observe that when $\alpha(t)$ is inserted into the loop, we have

$$1 + \alpha M = 1 + M + (\alpha - 1)M,$$

with $|\alpha|1| < 1$. It follows that

$$(1 + \alpha M)^{-1} = (1 + M + (\alpha - 1)M)^{-1}$$

$$= (1 + M)^{-1}(1 + (\alpha - 1)M(1 + M)^{-1})^{-1};$$

$$M(1 + \alpha M)^{-1} = M(1 + M)^{-1}(1 + (\alpha - 1)M(1 + M)^{-1})^{-1}.$$

By the Small Gain Theorem [18,19] and the fact that $|\alpha - 1| < 1$, the closed-loop system will remain stable for all $0 < \alpha < 1$ if

$$\mu = \|M(1 + M)^{-1}\|_\infty \le 1.$$

□

It should be pointed out that in the extreme case of $\alpha = 1$ stability is guaranteed. However, the stability of the system may be lost when $\alpha = 0$, indicating that control actions based solely on fairness coordination is not desirable. The conditions of Theorem 1 can be used to redesign F_i so that coordination for fairness will not destabilize the overall system. The redesign of F_i can be done by classical lead–lag design on Bode plots or by H^∞ design for optimal solutions.

4.2. Convergence to fairness

Asymptotic fairness requires that the rates at the switches reach their fair share values when network traffic patterns settle to their steady states. More precisely, assume that

$$\lim_{t\to\infty} B(t) = B, \qquad \lim_{t\to\infty} c_i(t) = c_i, \qquad i = 1, \ldots, m.$$

Then by Eq. (1),

$$\lim_{t\to\infty} a(t) = \frac{B - (c_1 + \cdots + c_m)}{n - m} := a.$$

Consequently, the limiting fairness is given by

$$f_i = c_i, \qquad i = 1, \ldots, m; \qquad f_i = a, \qquad i = m + 1, \ldots, n.$$

Theorem 2. *If*

1. F_i *is designed to satisfy the conditions of* Theorem 1;
2. $d_i(t)$ *is uniformly bounded*;
3. $\alpha(t) > 0$ *and*

$$\lim_{t\to\infty} \alpha(t) = 0,$$

then

$$\lim_{t\to\infty} x_i(t) = f_i.$$

Proof. Observe that the traffic management signal u_i^0 is given by

$$u_i^0 = -F_i e_i + K_i d_i = -F_i(M_1 d_i + M_2 f_i) + K_i d_i$$

$$= (-F_i M_1 + K_i)d_i - F_i M_2 f_i.$$

By the hypotheses and Theorem 1, the closed-loop system is robustly stable for any $0 < \alpha(t) < 1$ and $\tau_i \le \bar{\tau}_i$. Hence, both $-F_i M_1 + K_i$ and $F_i M_2$ are bounded. It follows that u_i^0 is uniformly bounded, since by hypothesis both $d_i(t)$ and $f_i(t)$ are uniformly bounded,

$$|u_i^0(t)| \le b < \infty.$$

As a result,

$$\lim_{t\to\infty} |\alpha(t) u_i^0(t)| \le b \lim_{t\to\infty} |\alpha(t)| = 0.$$

This implies, by Eq. (5), that

$$\lim_{t\to\infty} |u_i(t) - (1 - \alpha(t)) f_i(t)| = 0,$$

or

$$\lim_{t\to\infty} u_i(t) = \lim_{t\to\infty} f_i(t) = f_i,$$

since $\lim_{t\to\infty} \alpha(t) = 0$.

Finally, since $x_i(t) = u_i(t - \tau_i)$,

$$\lim_{t\to\infty} x_i(t) = \lim_{t\to\infty} u_i(t) = f_i,$$

namely, the rates become asymptotically fair. □

5. Algorithms

Based on the previous discussion, the ER for each connection at a switch will be generated by the following algorithm.

In this algorithm, we don't assume a fixed sampling interval. This will allow the model and analysis to be applicable to a heterogeneous network environment in which switches, connections, and software protocols have different speeds and sampling rates. As a result, the time interval $\Delta_i = t_i - t_{i-1}$ between consecutive time instances, at which the data is processed and ER is generated, may vary with time, among connections, and over switches.

Determining the actual delay time for ATM connections is not currently possible. Instead of determining the actual time delay of the connections, we propose instead to use a conservative estimate of the maximum time delay that a connection may experience. For control design, it is important that sufficient delay be incorporated into the algorithm to allow stability.

For a given *switch*, the following notation is used in the algorithm:

$n(t_i)$ The total number of connections at time t_i
$B(t_i)$ ABR of the switch at time t_i
τ_j Maximum round-trip time delay of the jth connection
δ_j The bottleneck threshold for the jth connection
$bottlenecked_j$ True if the jth connection is bottlenecked; false otherwise
$x_j(t_i)$ Measured input rate of the jth connection at time t_i
$d_j(t_i)$ The output rate of the jth connection at time t_i
$u_j(t_i)$ The ER of the jth connection at time t_i
$u_j^b(t_i)$ The feedback action of the jth connection at time t_i
$u_j^f(t_i)$ The feedforward action of the jth connection at time t_i
$m(t_i)$ The number of bottlenecked connections at time t_i
$M(t_i)$ Bandwidth used by the bottlenecked connections at time t_i
$q_j(t_i)$ The queue size of the jth connection at time t_i
q_j^0 The desired queue size of the jth connection
$f_j(t_i)$ The fairness rate of the jth connection at time t_i

Test for bottlenecked connection:

$m(t_i) = 0$; $M(t_i) = 0$,
for each connection j

$$t_i^j = \max\{t_k : t_k \le t_i - \tau_j\},$$

if $x_j(t_i) \le u_j(t_i) - \delta_j$ then
$bottlenecked_j = \text{TRUE}$

$$m(t_i) = m(t_i) + 1$$

$$M(t_i) = M(t_i) + x_j(t_i),$$

else
$bottlenecked_j = \text{FALSE}$
endif
endfor

Fairness rate calculation:

if $m(t_i) \ne n(t_i)$ then /* If some connections are not bottlenecked.*/
$a(t_i) = \dfrac{B(t_i) - M(t_i)}{n(t_i) - m(t_i)}$
endif
for each connection j
if $bottlenecked_j$ then
$f_j(t_i) = x_j(t_i)$ /* Fair rate is input rate */
else
$f_j(t_i) = a(t_i)$
endif
endfor

Robust control actions:

We will use the special case of a first-order feedback $F_j(s) = (k_p)/(Ts + 1)$ and a proportional feedforward $K_j = k$ for the algorithm. More complicated controllers can be similarly treated.

$$e_j(t_i) = q_j(t_i) - q_j^0$$

$$u_j^{\mathrm{b}}(t_i) = \frac{(k_p e_j(t_i) - u_j(t_{i-1}))\Delta_i}{T} + u_j(t_{i-1})$$

$$u_j^{\mathrm{f}}(t_i) = kd_j(t_i)$$

$$u_j(t_i) = u_j^{\mathrm{f}}(t_i) - u_j^{\mathrm{b}}(t_i).$$

Calculation of coordination coefficient:

We will use the traffic indicator (4) with $a > 0$, $b_2 = c_2 > 0$, and b_1, $c_1 > 0$. Hence,

$$g_j(t_i) = \frac{s}{s + b_2}(b_1 d_j(t_i) + c_1 B(t_i)).$$

It is implemented as

$$g_j(t_i) = b_1(d_j(t_i) - d_j(t_{i-1})) + c_1(B(t_i) - B(t_{i-1})) - b_2(g_j(t_{i-1})\Delta_i + g_j(t_{i-1}))$$

$$\alpha(t_i) = \frac{|g_j(t_i)|}{a + |g_j(t_i)|}.$$

Calculation of the ER:

for each connection j

$$ER_j = u_j(t_i) = \alpha(t_i)u_j(t_i) + (1 - \alpha(t_i))f_j(t_i)$$

endfor
Upon receipt of a BRM, update the ER field in the BRM cell with:

$$\mathrm{ER} = \max(\min(\mathrm{PCR}, \mathrm{ER}, \mathrm{ER}_j), \mathrm{MCR}).$$

6. Simulation results

To demonstrate the design methodologies introduced in this paper, we perform simulations on a network with four switches and four sources as shown in Fig. 4. Matlab/Simulink is used to simulate the switch dynamics and delays on connections. To understand the robustness of the controlled system, the connections in Fig. 4 have different time delays.

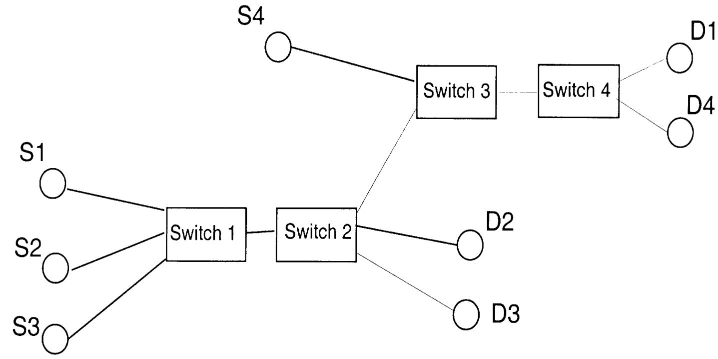

Fig. 4. An example of ABR network.

The exact values of time delays are not used in design. Rather their upper bounds are used to design controllers which can provide robustness against all possible delays within the bounds. For example, we assume that the actual delay time of the connection between source 1 and switch 1 is bounded by 0.1 s, which is used to design the feedback controllers.

In this simulation, for each connection i, F_i is a gain feedback $F_i = K_p$. It is easy to verify that if the maximum time delay for the connection is $\bar{\tau}_i$, then $K_p < \pi/2\bar{\tau}_i$ will guarantee robust stability for all $\tau_i \le \bar{\tau}_i$ without fairness coordination. However, to satisfy the condition of Theorem 1,

$$\sup_{0<\tau_i\le\bar{\tau}_i} \|M(1+M)^{-1}\|_\infty \le 1, \tag{6}$$

the gain must be further confined.

For example, for the maximum delay of $\bar{\tau}_i = 0.1$, robust stability against delay uncertainty requires that $K_p <$ 15.708. On the other hand, Fig. 5 illustrates the magnitudes of $M/(1+M)$ when $K_p = 10$. It shows $\| M(1+M)^{-1} \|_\infty = 2.327$, violating the condition of Theorem 1. By reducing K_p to 5, the norm condition (6) is satisfied as shown in Fig. 6.

For the selected K_p, feedforward $K_i(s)$ is to be designed to reduce

$$W(s)\frac{\mathrm{e}^{-\tau_i s}K_i(s) - 1}{1 + \frac{1}{s}\,\mathrm{e}^{-\tau_i s}K_p} = \frac{W(s)\,\mathrm{e}^{-\tau_i s}}{1 + \frac{1}{s}\,\mathrm{e}^{-\tau_i s}K_p}(K_i(s) - \mathrm{e}^{-\tau_i s})$$

$$= W_0(s)(K_i(s) - \mathrm{e}^{-\tau_i s}) \tag{7}$$

Since $|W_0(j\omega)|$ is very small outside $[0,\omega_M]$, reduction of Eq. (7) amounts essentially to approximation of $\mathrm{e}^{-\tau_i s}$ by a causal $K_i(s)$ in the frequency band $[0, \omega_M]$. Although the problem can be solved optimally by H^∞ optimization, the optimal solution will be an infinitely dimensional controller. To obtain a solution of low-order K_i, we will perform parametric optimization for simple K_i.

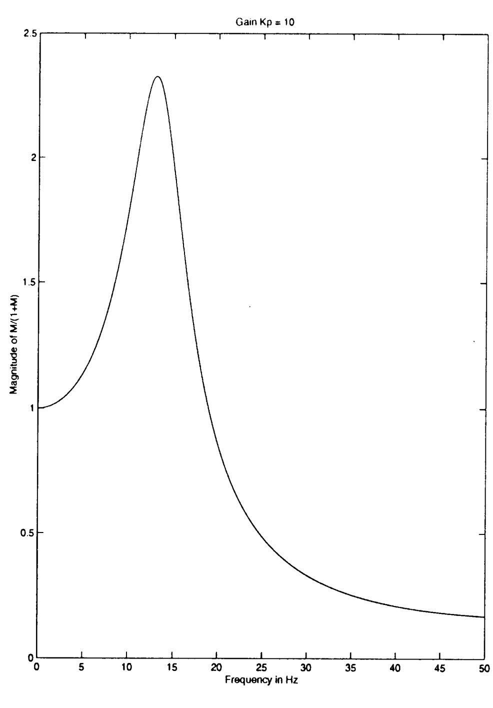

Fig. 5. Magnitude plot of $M/(1+M)$ for large K_p.

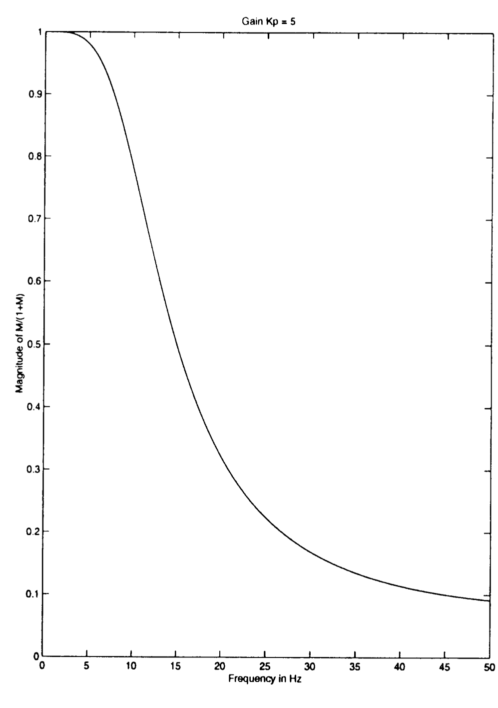

Fig. 6. Magnitude plot of $M/(1+M)$ for small K_p.

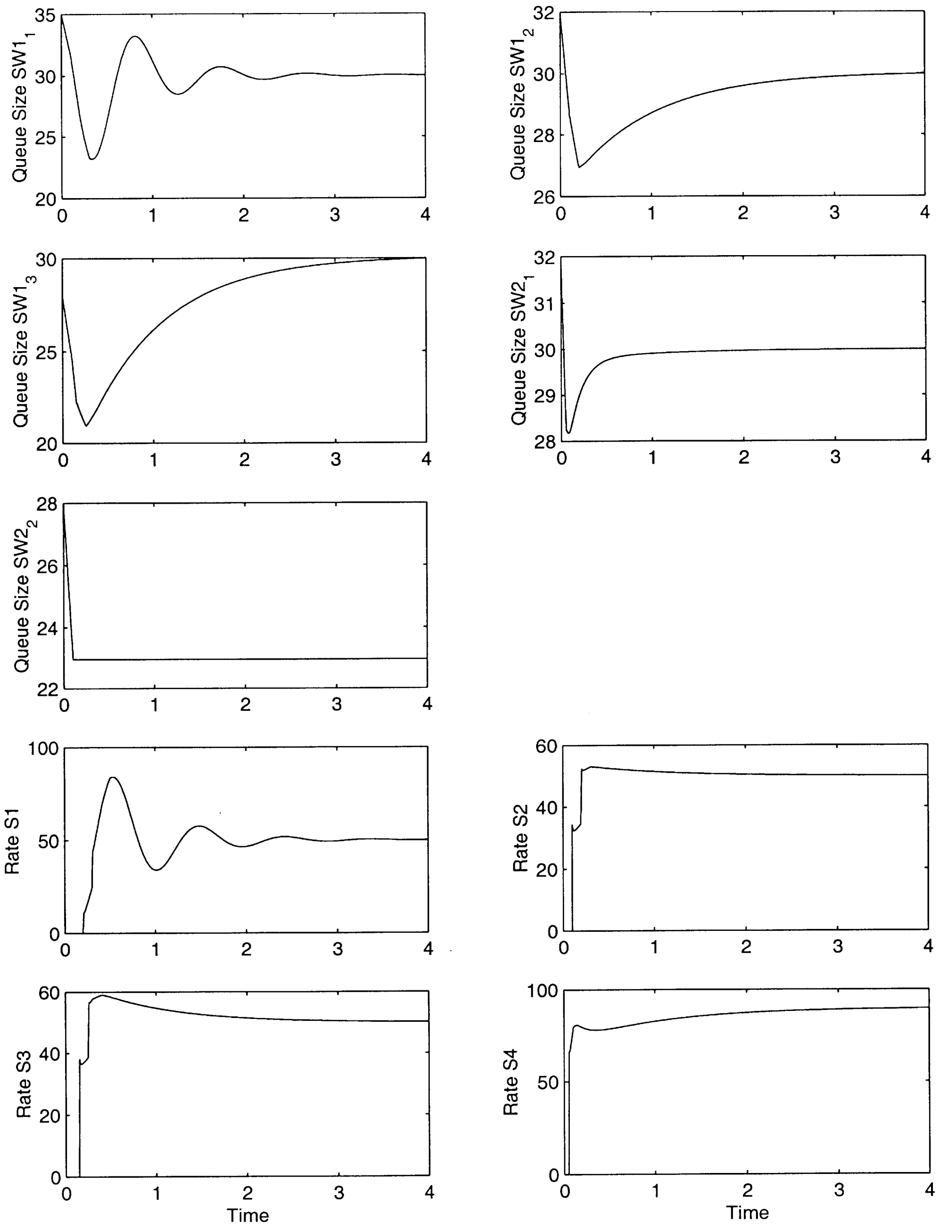

Fig. 7. Simulation for stability and fairness (rate: K cells/s; time: s).

For instance, let $\omega_M = 20$ (radian/s) and $\bar{\tau}_i\omega_M < \pi/2$.

1.

$K_i(s) = k.$

In this case, we consider

$$\max_{\omega\in[0,\omega_M]} |k - e^{j\bar{\tau}_i\omega}| = \max_{\omega\in[0,\omega_M]} ((k - \cos\bar{\tau}_i\omega)^2 + (\sin\bar{\tau}_i\omega)^2)^{1/2}$$

$$= \max_{\omega\in[0,\omega_M]} (k^2 - 2k\cos\bar{\tau}_i\omega + 1)^{1/2}$$

$$= (k^2 - 2k\cos\bar{\tau}_i\omega_M + 1)^{1/2}.$$

Hence, k can be selected by

$$\min_k (k^2 - 2k\cos\bar{\tau}_i\omega_M + 1),$$

which is solved as

$$k = \cos\bar{\tau}_i\omega_M.$$

For $\bar{\tau}_i = 0.05$ and $\omega_M = 20$, $k = 0.5403$.

2. $K_i = k((s+a)/(s+b))$, $k > 0$, $a > 0$, $b > 0$. Similar to the first case, we are seeking

$$\min_{k>0,a>0,b>0} \max_{\omega\in[0,\omega_M]} \left| k\frac{j\omega + a}{j\omega + b} - e^{j\bar{\tau}_i\omega} \right|.$$

A computer search can be performed to find the optimal parameter values.

In this simulation, we assume that the ABR for the connection between Switch 1 and Switch 2 initially varies with time and eventually settles to a steady state value of 150 (K cells/s). Similarly, the ABR for the connection between Switch 3 and Switch 4 eventually settles to 140. It is easy to check that at the steady state, the fairness is achieved if the source rates are $S1 = 50$, $S2 = 50$, $S3 = 50$, $S4 = 90$. The threshold for all queues is set to 30 and initial queue sizes are set to values different from their thresholds.

Simulation results are presented in Fig. 7. It is observed that under the control strategies introduced in this paper and after a stable dynamic transient process, the transmission rates of sources converge to their fair share values. The queue sizes also converge to their threshold of 30 cells.

Appendix A. A List of symbols

Numerous symbols are used in this paper and are summarized in the following list for quick reference.

S_i	Source i
SW_j	Switch j
D_k	Destination k
$r_i(t)$	Transmission rate of source i at time t
$x_i(t)$	Receiving rate of the switch for connection i at time t
$d_i(t)$	Output rate from the switch for connection i at time t
$q_i(t)$	Queue size of the switch buffer at time t
$e_i(t)$	Difference between the queue size and its desired value
$P_i(s)$	Transfer function of the linear time invariant plant i (the system from u_i to e_i)
$F_j(s)$	Transfer function of the linear time invariant controller j
M	Time varying closed-loop system
$u_i^0(t)$	Control action derived from queue control for source i
$f_i(t)$	Control action derived from fairness coordination for source i
$u_i(t)$	Total combined control action

References

[1] E. Altman, T. Basar, Multi-user rate-based flow control: distributed game-theoretic algorithms, 36th CDC Conference, December 1997, pp. 2916–2921.

[2] The ATM Forum, ATM Forum Traffic Management Specification Version 4.1, March 1999.

[3] F. Bonomi, D. Mitra, J.B. Seery, Adaptive algorithms for feedback-based flow control in high speed wide area ATM networks, IEEE Journal on Selected Areas in Communications 13 (7) (1995) 1267–1283.

[4] A. Charny, D. Clark, R. Jain, Congestion control with explicit rate indication, International Conference on Communications, 1995, pp. 1954–1963.

[5] F. Chiussi, Y. Xia, V.P. Kumar, Dynamic max rate control algorithm for available bit rate service in ATM networks, GLOBECOM'96, 1996.

[6] S. Fahmy et al., On determining the fair bandwidth share for ABR connections in ATM networks, International Conference on Communications, 1998.

[7] Y. Gong, I. Akyildiz, Dynamic traffic control using feedback and traffic prediction in ATM networks, IEEE INFOCOM'94, 1994, pp. 91–98.

[8] R. Jain, et al., The ERICA switch algorithm for ABR traffic management in ATM networks, IEEE/ACM Transactions on Networking 8 (1) (2000) 87–98.

[9] J. Jaffe, Bottleneck flow control, IEEE Transactions on Communications COM-29 (7) (1981) 954–962.

[10] S. Keshav, An Engineering Approach to Computer Networking: ATM Networks, the Internet, and the Telephone Network, Addison Wesley/Longman, Reading, MA/New York, 1997.

[11] A. Kolarov, G. Ramamurthy, A control-theoretic approach to the design of an explicit rate controller for ABR service, IEEE/ACM Transactions on Networking 7 (5) (1999) 741–753.

[12] R. Krishnan, Rate-based control schemes for ABR traffic — design principles and performance comparison, Computer Networks and ISDN Systems 29 (5) (1997) 583–593.

[13] S. Muddu et al., Max–min rate control algorithm for available bit rate service in ATM networks. International Conference on Communications, 1996, pp. 412–418.

[14] S. Pisanpattanakul, M. Yamamoto, H. Ikeda, A.B.R. Traffic, Control scheme for bursty traffic considering the number of hops, IEICE Transactions on Communications E81-B (11) (1998) 2056–2063.

[15] C.E. Rohrs, R.A. Berry, A linear control approach to explicit rate feedback in ATM networks. IEEE INFOCOM'97, 1997, pp. 277–282.

[16] K. Siu, T. Tzeng, Intelligent congestion control for ABR service in ATM networks, Computer Communication Review 24 (5) (1995) 81–106.

[17] P. Thomas, D. Teneketzis, An approach to service provisioning with quality of service requirements in ATM networks, 36th CDC Conference, December 1997, pp. 4568–4573.

[18] G. Vidyasagar, Nonlinear Systems Analysis, 2nd ed., Prentice-Hall, Englewood Cliffs, NJ, 1993.

[19] G. Zames, On the input–output stability of time-varying nonlinear feedback systems, IEEE Transactions on Automatic Control AC-11 (1966) 228–238 (see also 465–476).

[20] Y. Zhao, S.Q. Li, S. Sigarto, A. Linear, Dynamic model for design of stable explicit-rate ABR control schemes. IEEE INFOCOM'97, 1997, pp. 283–292.