Transformation and Viewing

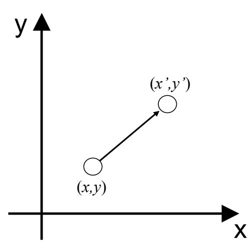
2D Geometric Transformations

- Translation
- Rotation
- Scaling
- Shear
- Homogenous Coordinates
- Matrix Representations
- Composite Transformations

Translation

- $x' = x + t_x$
- $y' = y + t_y$

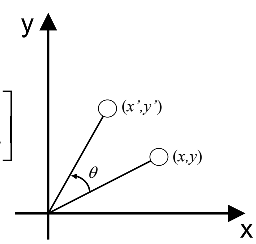
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$



Rotation

- $x' = x \cdot \cos\theta - y \cdot \sin\theta$
- $y' = x \cdot \sin\theta + y \cdot \cos\theta$

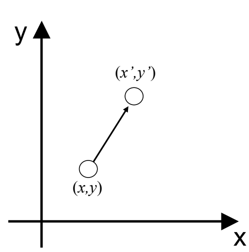
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



Scaling

- $x' = S_x \cdot x$
- $y' = S_y \cdot y$

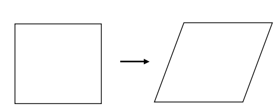
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



Shear

- $x' = x + h_x \cdot y$
- $y' = y$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & h_x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



Homogenous Coordinates

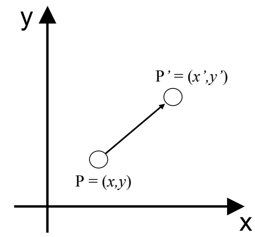
- Each position (x, y) is represented as $(x, y, 1)$.
- All transformations can be represented as matrix multiplication.
- Composite transformation becomes easier.

Translation in Homogenous Coordinates

- $x' = x + t_x$
- $y' = y + t_y$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\mathbf{P'} = \mathbf{T}(t_x , t_y) \bullet \mathbf{P}$$

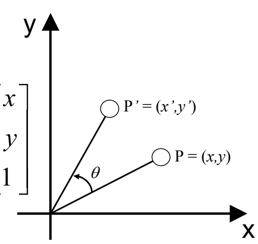


Rotation in Homogenous Coordinates

- $x' = x \cdot \cos\theta - y \cdot \sin\theta$
- $y' = x \cdot \sin\theta + y \cdot \cos\theta$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\mathbf{P'} = \mathbf{R}(\theta) \bullet \mathbf{P}$$

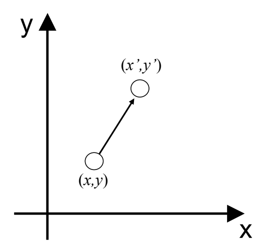


Scaling in Homogenous Coordinates

- $x' = s_x \cdot x$
- $y' = s_y \cdot y$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

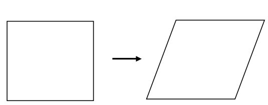
$$\mathbf{P'} = \mathbf{S}(s_x , s_y) \bullet \mathbf{P}$$



Shear in Homogenous Coordinates

- $x' = x + h_x \cdot y$
- $y' = y$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & h_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



$$\mathbf{P'} = \mathbf{SH}_x \bullet \mathbf{P}$$

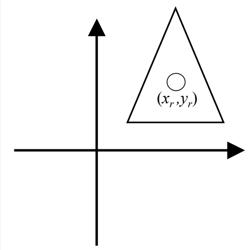
2D Geometric Transformations

- Translation
- Rotation
- Scaling
- Shear
- Homogenous Coordinates
- Composite Transformations

2D Geometric Transformations

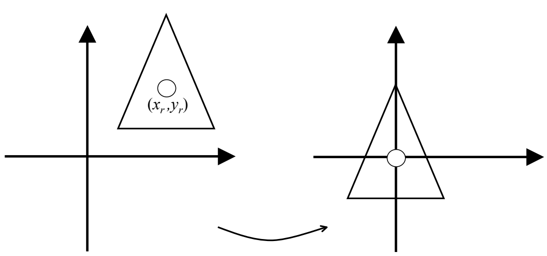
- Translation
- Rotation
- Scaling
- Shear
- Homogenous Coordinates
- Composite Transformations
  - Rotation about a fixed point

Rotation About a Fixed Point



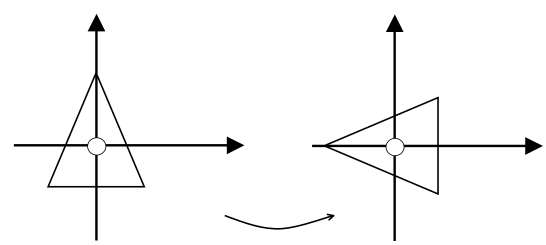
1. **Translate the object to the origin.**
2. **Rotate around the origin.**
3. **Translate the object back.**

Rotation About a Fixed Point



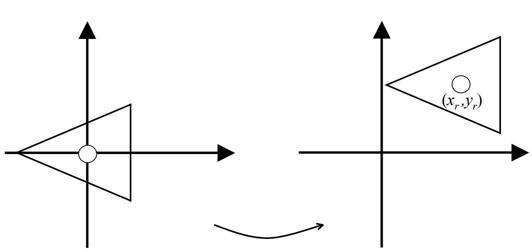
1. Translate the object to the origin

Rotation About a Fixed Point



2. Rotate about origin

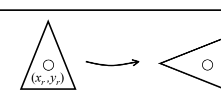
Rotation About a Fixed Point



3. Translate the object back

Rotation About a Fixed Point

1. **Translate the object to the origin.**
2. **Rotate around the origin.**
3. **Translate the object back.**



$$\begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}(x_r, y_r) \bullet \mathbf{R}(\theta) \bullet \mathbf{T}(-x_r, -y_r)$$

$$\mathbf{P}' = \mathbf{T}(x_r, y_r) \bullet \mathbf{R}(\theta) \bullet \mathbf{T}(-x_r, -y_r) \bullet \mathbf{P}$$

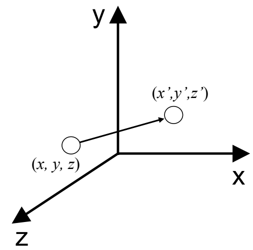
3D Geometric Transformations

- Basic 3D Transformations
  - Translation
  - Rotation
  - Scaling
  - Shear
- Composite 3D Transformations
- Change of Coordinate systems

Translation

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

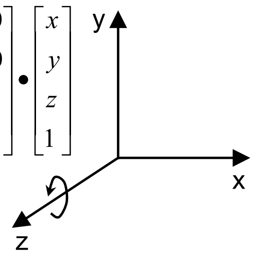
$$\mathbf{P' = T \cdot P}$$



Rotation about z-axis

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

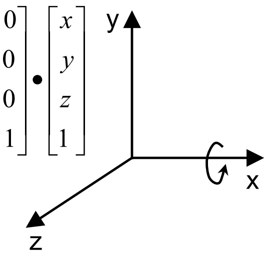
$$\mathbf{P' = R_z(\theta) \cdot P}$$



Rotation about x-axis

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

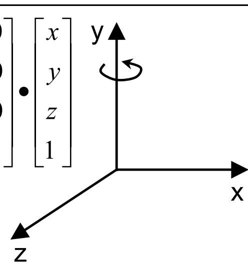
$$\mathbf{P' = R_x(\theta) \cdot P}$$



Rotation about y-axis

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\mathbf{P' = R_y(\theta) \cdot P}$$



Rotation About a Fixed Point

1. **Translate the object to the origin.**
2. **Rotate about the three axis, respectively.**
3. **Translate the object back.**

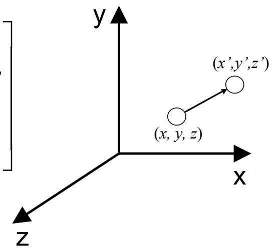
$$\mathbf{P' = T}\,(x_r, y_r, z_r) \bullet \mathbf{R} \bullet \mathbf{T}\,(-x_r, -y_r, -z_r) \bullet \mathbf{P}$$

$$\mathbf{R} = \mathbf{R_x}(\theta_x) \bullet \mathbf{R_y}(\theta_y) \bullet \mathbf{R_z}(\theta_z)$$

Scaling

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

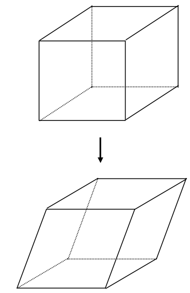
$$\mathbf{P'} = \mathbf{S} \bullet \mathbf{P}$$



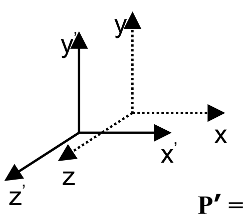
Shear

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & h_x & 0 \\ 0 & 1 & h_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

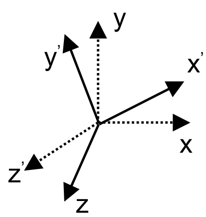
$$\mathbf{P'} = \mathbf{SH}_{xy} \bullet \mathbf{P}$$



Change in Coordinate Systems



$$\mathbf{P'} = \mathbf{M} \bullet \mathbf{P}$$



M can be a combination of translation, rotation and scaling.

Taking a picture with a camera

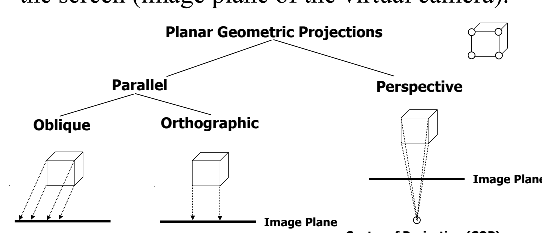
- Coordinates: Local, World, Viewing
- Rendering pipeline
- ModelView
  - Matrix operations on models
- World coordinates to Viewing coordinates
  - Matrix operations (models or cameras)
- Projection with a camera

Viewing in 3D

- Planar Geometric Projections
- Parallel Orthographic Projections
- Perspective Projections
- Projections in OpenGL
- Clipping

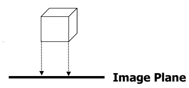
Planar Geometric Projections

- Maps points from camera coordinate system to the screen (image plane of the virtual camera).



Parallel Orthographic Projection

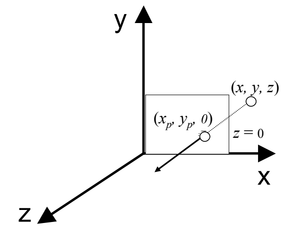
- Preserves X and Y coordinates.
- Preserves both distances and angles.



Parallel Orthographic Projection

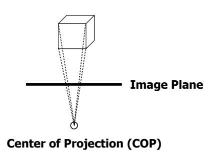
- $x_p = x$
- $y_p = y$
- $z_p = 0$

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

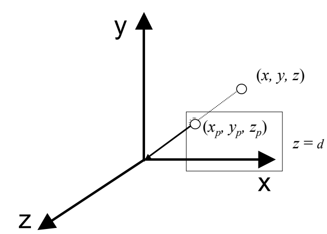


Perspective Projection

- Only preserves parallel lines that are parallel to the image plane.
- Line segments are shorten by distance.

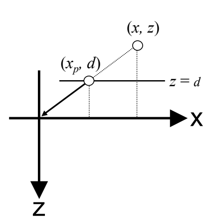


Perspective Projection



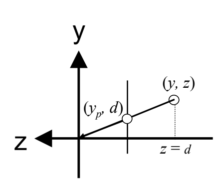
Perspective Projection

- $z_p = d$
- $x_p = (x \cdot d) / z$



Perspective Projection

- $z_p = d$
- $y_p = (y \cdot d) / z$



Perspective Projection

- $x_p = (x \cdot d) / z = x/(z/d)$
- $y_p = (y \cdot d) / z = y/(z/d)$
- $z_p = d \qquad = z/(z/d)$

Viewing in 3D

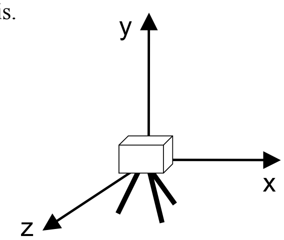
- Planar Geometric Projections
- Parallel Orthographic Projections
- Perspective Projections
- Projections in OpenGL

Viewing in 3D

- Planar Geometric Projections
- Parallel Orthographic Projections
- Perspective Projections
- Projections in OpenGL
  - Positioning of the Camera
  - Define the view volume

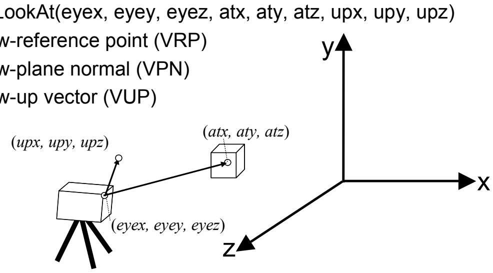
Positioning the Camera

- By default, the camera is placed at the origin pointing in the negative z-axis.



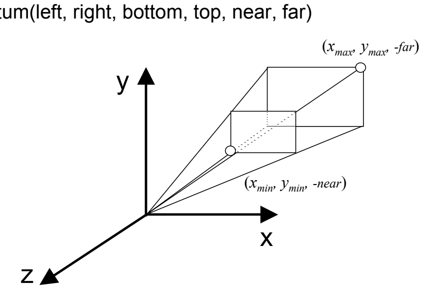
Positioning the Camera

- OpenGL Look-At Function
  gluLookAt(eyex, eyey, eyez, atx, aty, atz, upx, upy, upz)
- View-reference point (VRP)
- View-plane normal (VPN)
- View-up vector (VUP)



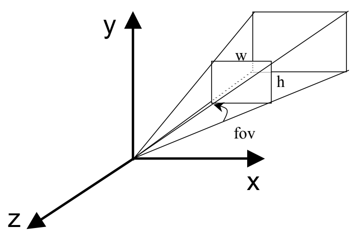
Defining the Perspective View Volume

glFrustum(left, right, bottom, top, near, far)



Defining the Perspective View Volume

gluPerspective(fovy, aspect, near, far)



Defining the Parallel View Volume

glOrtho(xmin, xmax, ymin, ymax, near, far)