Points and Particles

What is a particle?

- Particles are objects that have mass, position, velocity, acceleration, and other attributes.

- Particle systems consist of a large number of particles moving under the influence of external forces such as gravity, and collisions with stationary obstacles.

What is a particle?

- Particles can be made to exhibit a wide range of interesting behavior such as join, split, stretch, or simulating natural phenomena.

- There are mainly two types of particle systems,
  - Interacting particle systems. (mass-spring; molecule)
  - Non-interacting particles.

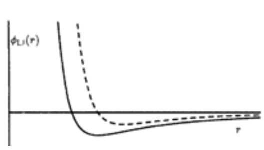
Interacting Particle Systems

- Ideas from molecular dynamics have been used to develop models of deformable materials using collections of interacting particles.

- In these models, long-range attraction forces and short-range repulsion forces control the dynamics of the system.

Interacting Particle Systems

- Typically, these forces are derived from an inter-molecular potential function such as the Lennard-Jones function:



Lennard-Jones type function: the solid line shows the potential function $\phi_{LJ}(r)$, and the dashed line shows the force function : $f(r) = -\frac{d}{dr}\phi_{LJ}(r)$, r is the distance between two particles.

Lennard-Jones functions

$$\phi_{LJ}(r) = \frac{-\epsilon}{n-m}\left(m\left(\frac{r_o}{r}\right)^n - n\left(\frac{r_o}{r}\right)^m\right),$$

Time Integration

- Given the initial position $x_i(t_0)$ and the initial velocity $v_i(t_0)$, we simulate Newtonian dynamics

$$a_i = \frac{f_i}{m_i}$$

$$a_i = \frac{dv_i}{dt}$$

$$v_i = \frac{dx_i}{dt}$$

- Time integration

$$t_{j+1} = t_j + \Delta t$$

- Sum of forces
  - inter-particle
  - collision
  - gravity
  - damping

Interacting Particle Systems

- Particle systems whose dynamics are governed by potential functions and damping will evolve towards lower energy states.
- In 3D the particles will arrange themselves into hexagonally ordered layers. They are naturally used to model solid objects via applied external forces.

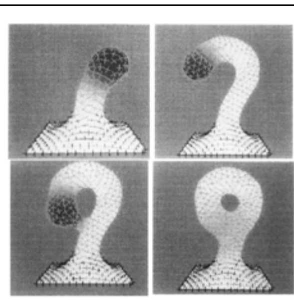
Interacting Particle Systems

- However, it is rather hard to model surface using particle systems.
- Since in the absence of external forces and constraints, 3D particle systems prefer to arrange themselves into solids rather than surfaces.

Oriented Particle Systems

- Szeliski and Tonnesen introduced oriented particle systems to model more flexible surfaces.
- They add an orientation to each particle's state and devise new interaction potentials for the oriented particles which favor locally planar or spherical arrangements.

Oriented Particle Systems



Non-interacting Particles

- Non-interacting particle systems have been used to model visually complex natural phenomena such as fire, smoke, foliage, and the spray of splashing water.
- In these systems, the Navier-Stokes equations are employed to describe the motion.

Navier-Stokes Equations

$$\nabla \cdot \mathbf{u} = 0,$$
$$\frac{\partial \mathbf{u}}{\partial t} = v\nabla \cdot (\nabla \mathbf{u}) - (u \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \mathbf{f},$$

- Where u is the velocity field, $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$
- The first equation conserves mass of the modeled object.

Navier-Stokes Equations

$$\nabla \cdot \mathbf{u} = 0,$$
$$\frac{\partial \mathbf{u}}{\partial t} = v\nabla \cdot (\nabla \mathbf{u}) - (u \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \mathbf{f},$$

- The second equation models the changes in the velocity field over time due to the effects of viscosity (v), convection, density(ρ), pressure(p), and external force (f).

Navier-Stokes Equations

- Solving the two equations can create a simulation of natural phenomena.
- The general framework for simulation or animation process can be described as follows:

Navier-Stokes Equations

For each simulation time step:

1. Update the velocity field by solving Equation 2 using numerical methods.
2. Apply velocity constraints due to obstacles.
3. Enforce mass conservation by solving a linear system build from Equation 1.
4. Update the position of the particles using the new velocity field.

Rendering

- Implicit Functions.
- Direct rendering
  – ray tracing

Rendering

- Implicit Functions.
  – Define a field function with each particle based on the distance to the particle.
  – The surface of the particle systems is all the points in space where the summation of all the individual field function equation to a threshold.

$$G(x, y, z) = \sum_i g_i(x, y, z) - threshold = 0$$

  – Where g_i is the field function for particle i.