Points and Particles

What is a particle?
• Particles are objects that have mass, position, velocity, acceleration, and other attributes.
• Particle systems consist of a large number of particles moving under the influence of external forces such as gravity, and collisions with stationary obstacles.

What is a particle?
• Particles can be made to exhibit a wide range of interesting behavior such as join, split, stretch, or simulating natural phenomena.
• There are mainly two types of particle systems,
  – Interacting particle systems. (mass-spring; molecule)
  – Non-interacting particles.

Interacting Particle Systems
• Ideas from molecular dynamics have been used to develop models of deformable materials using collections of interacting particles.
• In these models, long-range attraction forces and short-range repulsion forces control the dynamics of the system.

Interacting Particle Systems
• Typically, these forces are derived from an intermolecular potential function such as the Lennard-Jones function:

\[
\phi_{LJ}(r) = \frac{-C}{n-m} \left( \frac{m}{r} \right)^6 - \frac{a}{r} \left( \frac{P_0}{r} \right)^m,
\]

Lennard-Jones functions

Lennard-Jones type function: the solid line shows the potential function \(\phi_{LJ}(r)\), and the dashed line shows the force function \(f = -\frac{d\phi_{LJ}}{dr}\), \(r\) is the distance between two particles.
Time Integration

- Given the initial position $x_0(t_0)$ and the initial velocity $v_0(t_0)$, we simulate Newtonian dynamics:
  
  \[ a_i = \frac{F_i}{m_i} \]
  \[ v_{i+1} = v_i + \Delta t \cdot a_i \]
  \[ x_{i+1} = x_i + \Delta t \cdot v_{i+1} \]

- Time integration:

Interacting Particle Systems

- Particle systems whose dynamics are governed by potential functions and damping will evolve towards lower energy states.

- In 3D the particles will arrange themselves into hexagonally ordered layers. They are naturally used to model solid objects via applied external forces.

Interacting Particle Systems

- However, it is rather hard to model surface using particle systems.

- Since in the absence of external forces and constraints, 3D particle systems prefer to arrange themselves into solids rather than surfaces.

Oriented Particle Systems

- Szeliski and Tonnesen introduced oriented particle systems to model more flexible surfaces.

- They add an orientation to each particle’s state and devise new interaction potentials for the oriented particles which favor locally planar or spherical arrangements.

Oriented Particle Systems

Non-interacting Particles

- Non-interacting particle systems have been used to model visually complex natural phenomena such as fire, smoke, foliage, and the spray of splashing water.

- In these systems, the Navier-Stokes equations are employed to describe the motion.
Navier-Stokes Equations

\[ \nabla \cdot \mathbf{u} = 0, \]
\[ \frac{\partial \mathbf{u}}{\partial t} = \nu \nabla \cdot (\nabla \mathbf{u}) - (\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho} \nabla p + \mathbf{f}, \]

- Where \( \mathbf{u} \) is the velocity field, \( \nabla = (\partial / \partial x; \partial / \partial y; \partial / \partial z) \)
- The first equation conserves mass of the modeled object.

- The second equation models the changes in the velocity field over time due to the effects of viscosity (\( \nu \)), convection, density (\( \rho \)), pressure (\( p \)), and external force (\( \mathbf{f} \)).

Solving the two equations can create a simulation of natural phenomena. The general framework for simulation or animation process can be described as follows:

1. Update the velocity field by solving Equation 2 using numerical methods.
2. Apply velocity constraints due to obstacles.
3. Enforce mass conservation by solving a linear system build from Equation 1.
4. Update the position of the particles using the new velocity field.

Rendering

- Implicit Functions.
- Direct rendering
  - ray tracing

• Implicit Functions.
  – Define a field function with each particle based on the distance to the particle.
  – The surface of the particle systems is all the points in space where the summation of all the individual field function equation to a threshold.

\[ G(x, y, z) = \sum g_i(x, y, z) - \text{threshold} = 0 \]

- Where \( g_i \) is the field function for particle \( i \).