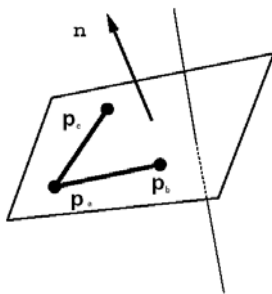


Surfaces

Surfaces

- Plane
- Quadratic surfaces
- Tensor product surfaces.
- Surfaces of revolutions.
- Sweeping surfaces.
- Subdivision surfaces.

Plane and Intersection



Plane Equations

- General plane equation
 $ax+by+cz+d=0$
- Normal of the plane
 $n=[a, b, c]$

Quadratic Surfaces

- Implicit representation
 $a_0x^2+a_1y^2+a_2z^2+a_3xy+a_4xz+a_5yz+a_6x+a_7y+a_8z+a_9=0$
- Sphere
 $x^2+y^2+z^2-r^2=0$
- Ellipsoid
 $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1=0$
- Torus:

Tensor Product Surface

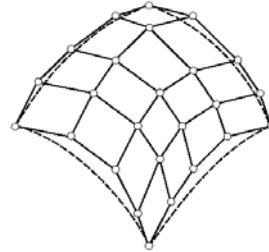
- From curves to surfaces
- A simple curve example (Bezier)
$$c(u) = \sum_{i=0}^3 p_i B_i(u)$$
where $u \in [0,1]$
- Consider p_i is a curve $p_i(v)$
- In particular, if p_i is also a bezier curve, where $v \in [0,1]$
$$P_i = \sum_{j=0}^3 p_{i,j} B_j(v)$$

From curve to surface

- Then we have

$$s(u, v) = \sum_{i=0}^3 \left(\sum_{j=0}^3 p_{i,j} B_j(v) \right) B_i(u) = \sum_{i=0}^3 \sum_{j=0}^3 p_{i,j} B_i(u) B_j(v)$$

Bezier Surface



B-Splines Surface

- B-Spline curves

$$c(u) = \sum_{i=0}^n p_i B_{i,k}(u)$$

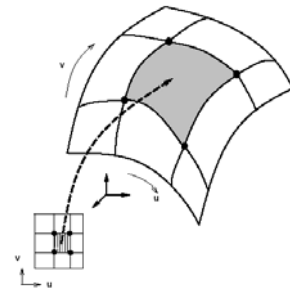
- Tensor product B-splines

$$s(u, v) = \sum_{i=0}^m \sum_{j=0}^n p_{i,j} B_{i,k}(u) B_{j,l}(v)$$

where $u \in [0,1]$, and $v \in [0,1]$

- Can we get NURBS surface this way?

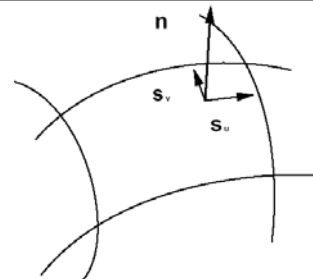
B-Splines Surface



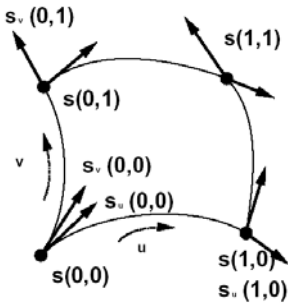
Tensor Product Properties

- Inherit from their curve generators.
- Continuity across boundaries
- Interpolation and approximation tools.

Surface Normal



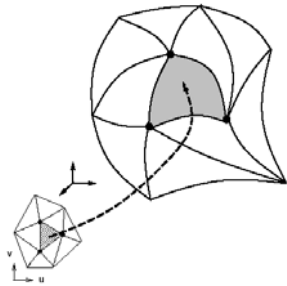
Hermite Surfaces



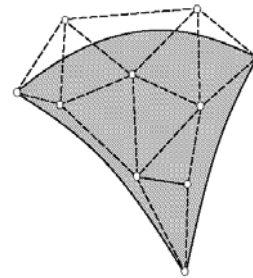
NURBS Surface

$$s(u, v) = \frac{\sum_{i=0}^n \sum_{j=0}^m P_{i,j} w_{i,j} B_{i,k}(u) B_{j,l}(v)}{\sum_{i=0}^n \sum_{j=0}^m w_{i,j} B_{i,k}(u) B_{j,l}(v)}$$

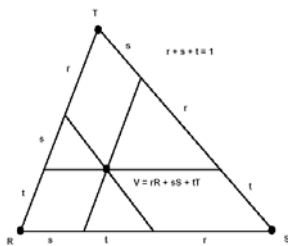
Triangular Surfaces



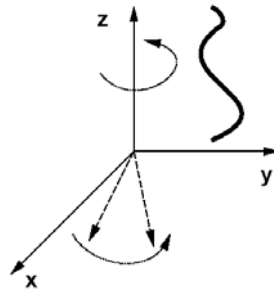
Triangular Bezier Surface



Barycentric Coordinates



Surface of Revolution



Surface of Revolution

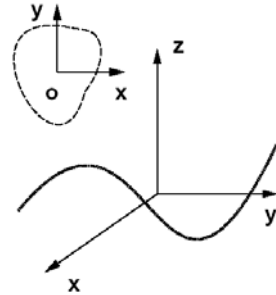
- Geometric construction
 - Specify a planar curve profile on y-z plane
 - Rotate this profile with respect to z-axis
- Procedure-based model

$$c(u) = \begin{bmatrix} 0 \\ y(u) \\ z(u) \end{bmatrix}$$
$$s(u, v) = \begin{bmatrix} -y(u) \sin v \\ y(u) \cos v \\ z(u) \end{bmatrix}$$

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Sweeping Surface



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Sweeping Surface

- Surface of revolution is a special case of a sweeping surface.
- Idea: a profile curve and a trajectory curve.
- Move a profile curve along a trajectory curve to generate a sweeping surface.

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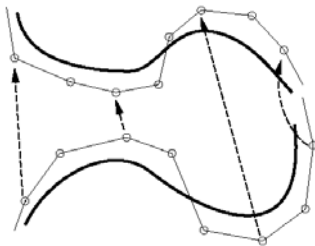
Geometric Modeling Techniques

- Control Point Manipulation.
- Weight Modification.
- Knot Vector Variation.
- Dynamic Modeling

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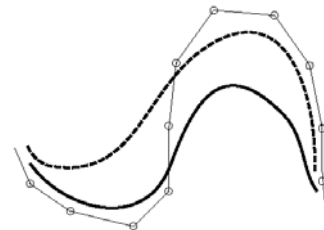
Control Point Manipulation



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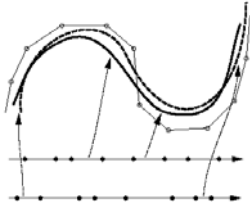
Weight Modification



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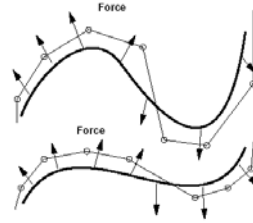
Knot Vector Variation



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Dynamic Modeling



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