Surfaces

- Plane
- Quadratic surfaces
- Tensor product surfaces.
- Surfaces of revolutions.
- Sweeping surfaces.
- Subdivision surfaces.

Plane and Intersection

- General plane equation
  \[ ax + by + cz + d = 0 \]
- Normal of the plane
  \[ n = [a, b, c] \]

Quadratic Surfaces

- Implicit representation
  \[ ax^2 + by^2 + cz^2 + ex + fy + gz + h = 0 \]
- Sphere
  \[ x^2 + y^2 + z^2 = r^2 \]
- Ellipsoid
  \[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \]
- Torus:

Tensor Product Surface

- From curves to surfaces
- A simple curve example (Bezier)
  \[ c(u) = \sum_{i=0}^{3} p_i B_i(u) \]
  where \( u \in [0,1] \)
- Consider \( p_i \) is a curve \( p_i(v) \)
- In particular, if \( p_i \) is also a bezier curve, where
  \( v \in [0,1] \)
  \[ p_i = \sum_{j=0}^{3} p_{ij} B_j(v) \]
From curve to surface

Then we have
\[
\mathbf{s}(u,v) = \sum_{i=0}^{3} \left( \sum_{j=0}^{3} p_{ij} B_j(v) R_i(u) \right) = \sum_{i=0}^{3} \sum_{j=0}^{3} p_{ij} B_j(v) R_i(u)
\]

B-Splines Surface

B-Spline curves
\[
c(u) = \sum_{i=0}^{n} p_i B_i(u)
\]

Tensor product B-splines
\[
\mathbf{s}(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} p_{ij} B_i(u) R_j(v)
\]

where \( u \in [0,1] \) and \( v \in [0,1] \)

• Can we get NURBS surface this way?

Tensor Product Properties

• Inherit from their curve generators.
• Continuity across boundaries
• Interpolation and approximation tools.

Surface Normal
Hermite Surfaces

NURBS Surface

\[ s(u, v) = \frac{\sum_{i=0}^{n} \sum_{j=0}^{m} P_{i,j} u_i v_j B_i(n) B_j(v)}{\sum_{i=0}^{n} \sum_{j=0}^{m} u_i v_j B_i(n) B_j(v)} \]

Triangular Surfaces

Triangular Bezier Surface

Barycentric Coordinates

Surface of Revolution
Surface of Revolution

- Geometric construction
  - Specify a planar curve profile on y-z plane
  - Rotate this profile with respect to z-axis
- Procedure-based model
  \[ c(u) = \begin{bmatrix} 0 \\ y(u) \\ z(u) \end{bmatrix} \]
  \[ s(u, v) = \begin{bmatrix} y(u) \sin v \\ y(u) \cos v \\ z(u) \end{bmatrix} \]

Sweeping Surface

- Surface of revolution is a special case of a sweeping surface.
- Idea: a profile curve and a trajectory curve.
- Move a profile curve along a trajectory curve to generate a sweeping surface.

Geometric Modeling Techniques

- Control Point Manipulation.
- Weight Modification.
- Knot Vector Variation.
- Dynamic Modeling

Control Point Manipulation

Weight Modification