2D Scan-line Conversion

2D Scan-line Conversion

- DDA algorithm
- Bresenham's algorithm

DDA algorithm

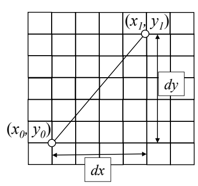
- The simplest algorithm.
- Named after Digital Differential Analyzer.

$$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{dy}{dx}$$

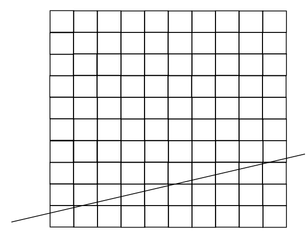
$$0 \le m \le 1$$

$$\Delta y = m\Delta x, \quad \Delta x = 1$$

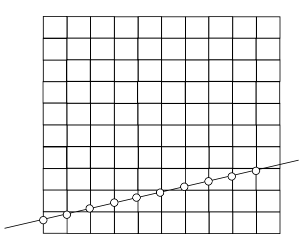
$$So, \Delta y = m$$



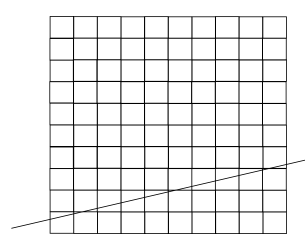
DDA Algorithm



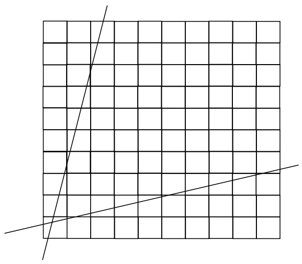
DDA Algorithm



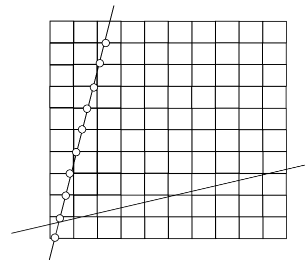
DDA Algorithm



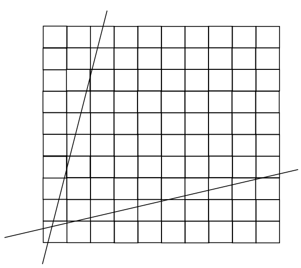
DDA Algorithm



DDA Algorithm



DDA Algorithm

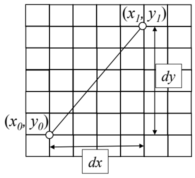


2D Scan-line Conversion

- DDA algorithm
- Bresenham's algorithm

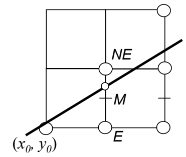
Bresenham's Midpoint Algorithm

- DDA is simple, efficient, but needs floating points.
- Bresenham's use integer addition only.



Bresenham's Midpoint Algorithm

- To choose from the two pixels *NE* or *E* depending on the relative position of the midpoint *M* and the line.
- Choose *E* if *M* is below the line,
- Choose *NE* if *M* is above the line.



Bresenham's Midpoint Algorithm

- Choose *NE* if *d* is negative,
- Choose *E* if *d* is positive.

Line equation :

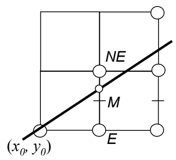
$$y = mx + b = \frac{dy}{dx}x + b$$

Implicit form :

$$F(x, y) = dy \cdot x - dx \cdot y + b \cdot dx = 0$$

Since point (x_0, y_0) is on the line, so

$$F(x_0, y_0) = dy \cdot x_0 - dx \cdot y_0 + b \cdot dx = 0$$



Bresenham's Midpoint Algorithm

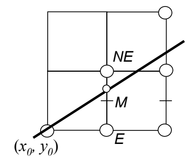
- Choose *NE* if *d* is negative,
- Choose *E* if *d* is positive.

The choice depends on the value of the decision variable d :

$$F(x_0 + 1, y_0 + \tfrac{1}{2}) = dy - dx \cdot \tfrac{1}{2}$$

$$2F(x_0 + 1, y_0 + \tfrac{1}{2}) = 2 \cdot dy - dx$$

$$d_1 = 2 \cdot dy - dx$$



Incremental Calculation of the decision variable d_{new}

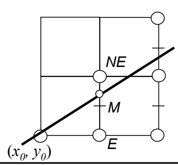
$Since \quad d = 2F(x, y) = 2dy \cdot x - 2dx \cdot y + 2b \cdot dx$

if choose E,

$$\begin{aligned} d_{new} &= 2F(x_0 + 2, y_0 + \tfrac{1}{2}) \\ &= 2dy \cdot (x_0 + 2) - 2dx \cdot (y_0 + \tfrac{1}{2}) + 2b \cdot dx \\ &= d_{old} + 2dy \end{aligned}$$

if choose NE,

$$\begin{aligned} d_{new} &= 2F(x_0 + 2, y_0 + \tfrac{3}{2}) \\ &= 2dy \cdot (x_0 + 2) - 2dx \cdot (y_0 + \tfrac{3}{2}) + 2b \cdot dx \\ &= d_{old} + 2dy - 2dx \end{aligned}$$



Bresenham's Midpoint Algorithm

$$d_{new} = \begin{cases} d_{old} + 2 \cdot \Delta y, & if \text{ choose } E \\ d_{old} + 2 \cdot \Delta y - 2 \cdot \Delta x, & f \text{ choose } NE \end{cases}$$