Geometrical Transformations

2D Geometrical Transformations

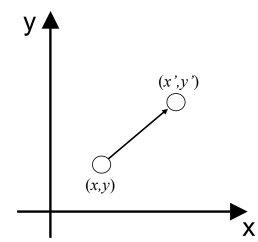
2D Geometric Transformations

- Translation
- Rotation
- Scaling
- Shear
- Homogenous Coordinates
- Matrix Representations
- Composite Transformations

Translation

- $x' = x + t_x$
- $y' = y + t_y$

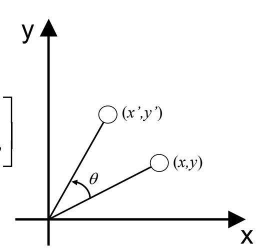
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$



Rotation

- $x' = x \cdot \cos\theta - y \cdot \sin\theta$
- $y' = x \cdot \sin\theta + y \cdot \cos\theta$

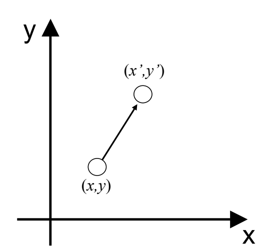
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



Scaling

- $x' = S_x \cdot x$
- $y' = S_y \cdot y$

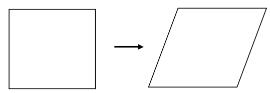
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



Shear

- $x' = x + h_x \cdot y$
- $y' = y$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & h_x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



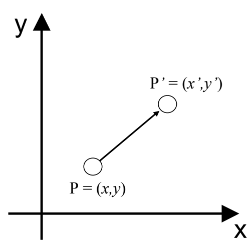
Homogenous Coordinates

- Each position (x, y) is represented as $(x, y, 1)$.
- All transformations can be represented as matrix multiplication.
- Composite transformation becomes easier.

Translation in Homogenous Coordinates

- $x' = x + t_x$
- $y' = y + t_y$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

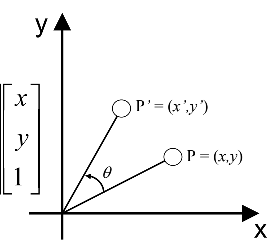


$\mathbf{P'} = \mathbf{T}(t_x, t_y) \cdot \mathbf{P}$

Rotation in Homogenous Coordinates

- $x' = x \cdot \cos\theta - y \cdot \sin\theta$
- $y' = x \cdot \sin\theta + y \cdot \cos\theta$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

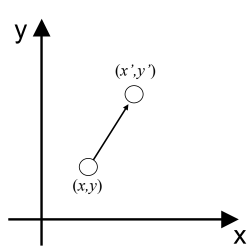


$\mathbf{P'} = \mathbf{R}(\theta) \cdot \mathbf{P}$

Scaling in Homogenous Coordinates

- $x' = s_x \cdot x$
- $y' = s_y \cdot y$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

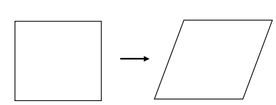


$\mathbf{P'} = \mathbf{S}(s_x, s_y) \cdot \mathbf{P}$

Shear in Homogenous Coordinates

- $x' = x + h_x \cdot y$
- $y' = y$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & h_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



$\mathbf{P'} = \mathbf{SH}_x \cdot \mathbf{P}$

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- Translation
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2D Geometric Transformations

- Translation
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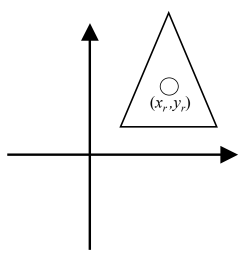
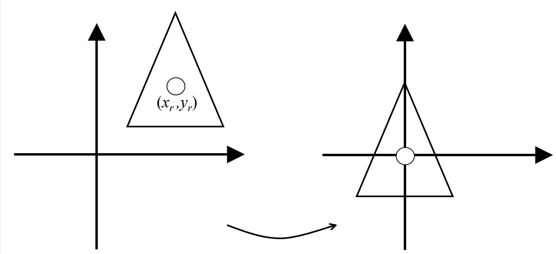
- Composite Transformations
  - Rotation about a fixed point

Rotation About a Fixed Point



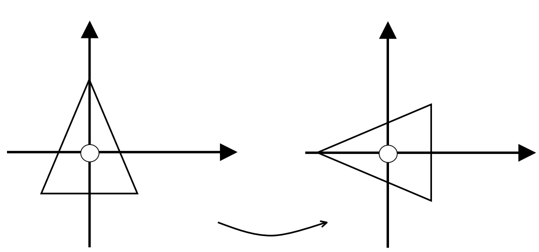
1. **Translate the object to the origin.**
2. **Rotate around the origin.**
3. **Translate the object back.**

Rotation About a Fixed Point



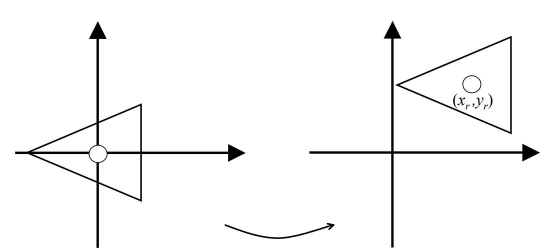
1. Translate the object to the origin

Rotation About a Fixed Point



2. Rotate about origin

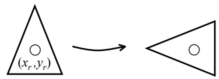
Rotation About a Fixed Point



3. Translate the object back

Rotation About a Fixed Point

1. **Translate the object to the origin.**
2. **Rotate around the origin.**
3. **Translate the object back.**



$$\begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}(x_r, y_r) \bullet \mathbf{R}(\theta) \bullet \mathbf{T}(-x_r, -y_r)$$

$$\mathbf{P'} = \mathbf{T}(x_r, y_r) \bullet \mathbf{R}(\theta) \bullet \mathbf{T}(-x_r, -y_r) \bullet \mathbf{P}$$

3D Geometrical Transformations

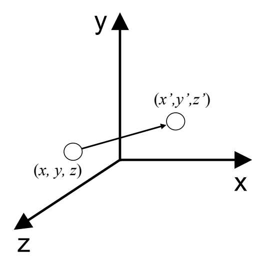
3D Geometric Transformations

- Basic 3D Transformations
  - Translation
  - Rotation
  - Scaling
  - Shear
- Composite 3D Transformations
- Change of Coordinate systems

Translation

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

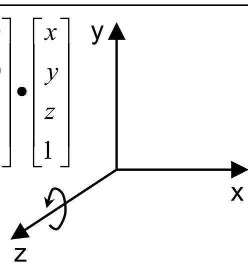
$$\mathbf{P'} = \mathbf{T} \bullet \mathbf{P}$$



Rotation about z-axis

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

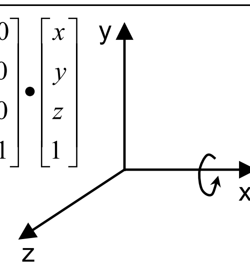
$$\mathbf{P'} = \mathbf{R_z}(\theta) \bullet \mathbf{P}$$



Rotation about x-axis

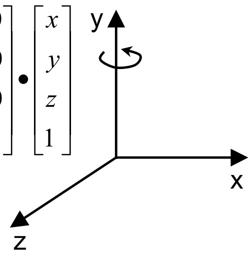
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\mathbf{P'} = \mathbf{R_x}(\theta) \bullet \mathbf{P}$$



Rotation about y-axis

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



$$\mathbf{P' = R_y(\theta) \cdot P}$$

Rotation About a Fixed Point

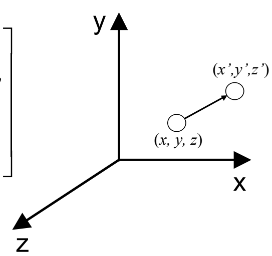
1. **Translate the object to the origin.**
2. **Rotate about the three axis, respectively.**
3. **Translate the object back.**

$$\mathbf{P' = T}\,(x_r, y_r, z_r) \bullet \mathbf{R} \bullet \mathbf{T}\,(-x_r, -y_r, -z_r) \bullet \mathbf{P}$$

$$\mathbf{R = R_x}(\theta_x) \bullet \mathbf{R_y}(\theta_y) \bullet \mathbf{R_z}(\theta_z)$$

Scaling

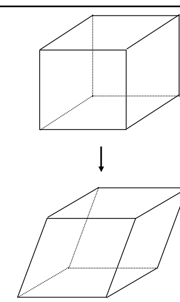
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



$$\mathbf{P' = S \bullet P}$$

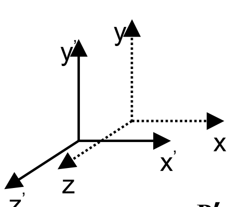
Shear

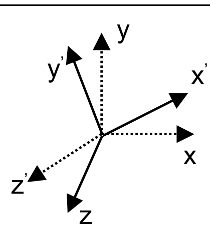
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & h_x & 0 \\ 0 & 1 & h_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



$$\mathbf{P' = SH}_{xy} \bullet \mathbf{P}$$

Change in Coordinate Systems





$$\mathbf{P' = M \bullet P}$$

M can be a combination of translation, rotation and scaling.