# CSC 6290 Midterm Take-home Exam

Fall 2008

Chapter 4:

1. exercise 4.4:

**4.4** A process $A$ with envelope $E_1$ is passed through a leaky bucket regulator with envelope $E_2$. Show that the resulting output has envelope $E_1 * E_2$.

**Solution:** Since the regulator is a leaky bucket, we have $D = A * E_2$. Since $A$ has envelope $E_1$, we further have $A \leq A * E_1$. Combining these observations, we have $D \leq (A*E_1)*E_2$ (in doing which, we have already used the property that $*$ distributes over min). Now use the associativity of $*$ and the fact that $E_2 = E_2*E_2$ to obtain $D \leq (A*E_2)*(E_1*E_2) = D*(E_1*E_2)$, i.e., $D$ has envelope $E_1 * E_2$.

2. Exercise 4.5:

**4.5** Consider a source with peak rate $R$ and packet size $L$, i.e., the packets are spaced by no less than $\frac{L}{R}$ seconds (e.g., a voice coder and packetiser emit 200 byte packets (160 bytes of PCM voice plus 40 bytes of RTP/UDP/IP headers) every 20ms, yielding $L = 200$ bytes and $R = 10$KBps). This source is shaped with a LB with parameters $(\sigma, \rho)$, with $\sigma \geq L$, and $\rho \leq R$. The source is served by a latency rate server with lower service curve that has rate $r$ and latency $d$ (or, a tandem of service elements that has this lower service curve). Show that:

a. the source has an envelope $(L + Rt)I_{\{t \geq 0\}}$,

b. the output of the LB has an envelope $\min((L + Rt)I_{\{t \geq 0\}}, (\sigma + \rho t)I_{\{t \geq 0\}})$, and

c. for $r < R$, the delay experienced by the source is bounded by

$$d + \left(\frac{\sigma - L}{r}\right)\left(\frac{R - r}{R - \rho}\right) + \frac{L}{r}$$

**Solution:**

a. Consider an interval $(t, t+\tau)$, with $\tau \geq 0$. Suppose a packet arrives at $t$; no more than $\frac{\tau}{(L/R)}$ additional packets can arrive over the remaining interval. Hence the amount of data emitted by the source over this interval is bounded by $L + L \times \frac{\tau}{(L/R)} = L + R\tau$.

b. Let us denote the source envelope by $A(t)$ and the LB envelope by $E(t)$. It then follows from Exercise 4.4 that the regulated output, $D(t)$, has envelope $(A * E)(t)$. Now we have

$$(A * E)(t) = \inf_{\tau \in \Re} f_\tau(t)$$

where, for all $t \in \Re$,

$$f_\tau(t) = \begin{cases} (\sigma + \rho(t - \tau))I_{(t-\tau \geq 0)} & \tau < 0 \\ (L + R\tau) + (\sigma + \rho(t - \tau))I_{(t-\tau \geq 0)} & \tau \geq 0 \end{cases}$$

It is then seen that $\inf_{\tau<0} f_\tau(t) = (\sigma + \rho(t))I_{(t\geq 0)}$. For $\tau \geq 0$, consider the two cases: (i) $\tau \leq t$, for which we have

$$\begin{aligned} f_\tau(t) &= (L + R\tau) + (\sigma + \rho(t - \tau)) \\ &\geq \sigma + \rho t \end{aligned}$$

since $R > \rho$, and (ii) $\tau > t$, for which we have

$$\begin{aligned} f_\tau(t) &= L + R\tau \\ &\geq L + Rt \end{aligned}$$

It follows that

$$(A * E)(t) \geq \min((L + Rt)I_{\{t\geq 0\}}, (\sigma + \rho t)I_{\{t\geq 0\}})$$

We already know that, for causal $A$ and $E$, $(A*E)(t) \leq \min\{A(t), E(t)\}$. Hence we conclude that

$$(A * E)(t) = \min((L + Rt)I_{\{t\geq 0\}}, (\sigma + \rho t)I_{\{t\geq 0\}})$$

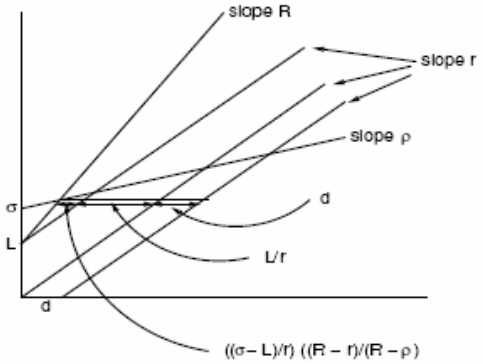


Figure 4.1: Answer to the third part of Exercise 4.5

c. The solution can be obtained geometrically and is clear from Figure 4.1